The Macroeconomics of Testing and Quarantining*†

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April 1, 2021

Abstract

We develop a SIR-based macroeconomic model to study the impact of testing/ quarantining and non-pharmaceutical interventions (NPIs) on health and economic outcomes. These policies can dramatically reduce the costs of an epidemic. Absent testing/quarantining, the main effect of NPIs on health outcomes is to delay, rather than reduce, epidemic-related deaths. NPIs reduce the severity of the epidemic-related recession but prolong its duration. There is an important synergy between NPIs and testing/quarantining. NPIs buy time for PIs to come to the rescue. The benefits of testing/quarantining are even larger when people can get reinfected, either because the virus mutates or immunity is temporary.

JEL Classification: E1, I1, H0
Keywords: Containment, COVID-19, epidemic, immunity, quarantine, testing.

*We thank Joao Guerreiro and Laura Murphy for excellent research assistance. We also are thank Andrew Atkeson, Chad Jones, Chris Limnios, and Pete Klenow for their comments.

†Matlab/Dynare replication codes are available at the following URL:
`https://sites.google.com/site/mathiastrabandt/home/research`

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1 Introduction

There are three obvious ways to deal with a new virus like the one that causes Covid: testing and quarantining infected people, developing vaccines, and discovering effective treatments. It takes considerable time and resources to develop and implement these "pharmaceutical interventions" (PIs). In contrast, non-pharmaceutical interventions (NPIs), like social distancing and mask use, are relatively inexpensive and can be quickly adopted. In this paper, we analyze the efficacy of different interventions and their interactions in a general equilibrium model of epidemics and economic activity. The NPIs that we consider are social distancing and mask use. The PIs that we focus on are testing and quarantining.[1]

Our main findings are as follows. First, absent PIs, the main effect of NPIs on health outcomes is to delay, rather than reduce, epidemic-related deaths. Second, with or without PIs, NPIs reduce the severity of the epidemic-related recession but prolong its duration.

Third, there is an important synergy between NPIs and PIs. In effect, NPIs buy time for PIs to come to the rescue. To understand this result, suppose that we fix the time it takes for PIs to arrive (say, one year). If NPIs are not widely adopted, then the virus spreads quickly. So, by the time PIs arrive, most of the deaths have already occurred. If NPIs are widely adopted, then the virus spreads slowly. So, in this scenario, NPIs avoid the bulk of deaths until PIs arrive and substantially reduce the death toll from the epidemic. Since people's market activities depend on the risk of infection, the synergies between NPIs and PIs also manifest themselves in economic outcomes.

The fourth finding is that the benefits of testing and quarantining are even larger when people can get reinfected, either because the virus mutates or immunity is temporary.

A central implication of our model is that if PIs never arrive, the effect of NPIs on overall deaths is quite small. The 1918 epidemic provides a natural test of this implication because PIs never arrived. Barro's (2020) finds that the effect of NPIs on overall deaths in the 1918 epidemic was small. So, Barro's (2020) evidence provides strong support for our model.

Our findings extend in a straightforward way to PIs like vaccines and treatments because there are clear benefits of buying time until they arrive. Eichenbaum, Rebelo and Trabandt (2020) study these benefits in a model in which vaccines and treatments arrive with a constant probability per period. In this framework, it is optimal to reduce economic activity in

[1] There is some ambiguity in the literature about whether testing and quarantining should be called a PI or a NPI. The ambiguity arises because testing involves the use of pharmaceuticals while quarantining does not. To simplify, we classify testing and quarantining as a PI.

anticipation of the arrival of vaccines or treatments. Makris and Toxvaerd (2020) study the benefits of buying time in a model where vaccines or treatments arrive at a known future date. In their framework, it is optimal for individuals to ramp up social distancing just before the arrival of NPIs.

In this paper, we focus on testing and quarantining in part because their interaction with NPIs is more subtle. Our results regarding health outcomes extend to any NPI that slows down the transmission of the virus, such as lockdowns. But the economic outcomes can be quite different. Lockdowns increase the severity of the recession. In contrast, mask use and social distancing mitigate the severity of the recession.[2]

Our analysis is based on a general equilibrium version of the Kermack and McKendrick (1927) SIR model. In our framework, people can get infected through consumption, work, and social activities. The model features a two-way interaction between the epidemic dynamics and economic activity. On the one hand, the epidemic causes a recession because, to reduce the chances of being infected, people cut back on forms of consumption and work that require social interaction. On the other hand, the fall in economic interactions slows down the transmission of the virus. We use a calibrated version of the model that accounts for the severity of the epidemic-related recession and the cumulative deaths from the epidemic.

Our paper is organized as follows. In Section 2, we describe our benchmark economy with NPIs but no testing and quarantining. In Section 3, we extend the framework to incorporate testing and quarantining. Section 4 presents our quantitative results. In Section 5, we consider a model in which people can become reinfected. Section 6 concludes.

2 Economy without testing and quarantining

This section is organized as follows. We first discuss the economy before the start of the epidemic. Then, we describe the competitive equilibrium without testing and quarantining.

[2]Mendoza et al. (2020) provide cross-country evidence on the negative impact of lockdowns on economic activity. Authors like Alvarez and Lippi (2020), Jones, Philippon, and Venkateswaran (2020), and Gonzalez-Eiras and Niepelt (2020) study models in which lockdowns increase the severity of the recession. Crucini and O'Flaherty (2020) study the impact of lockdowns in an macroeconomic SIR model with multiple locations. Kaplan, Moll, and Violante (2020) study the distributional consequences of alternative containment strategies. Bodenstein, Corsetti, and Guerrieri (2020) analyze the impact of lockdowns in models that have an input-output structure. They argue that the cost of such lockdowns is small if they prevent core sectors in the economy from becoming incapacitated.

2.1 The pre-epidemic economy

The economy is populated by a continuum of ex-ante identical people with measure one. The representative person maximizes the following objective function:

$$U = \sum_{t=0}^{\infty} \beta^t u(c_t, n_t, m_t).$$

Here, $\beta \in (0,1)$ denotes the discount factor and $m_t \in (0,1)$ denotes the level of NPIs. The variables c_t and n_t denote consumption and hours worked, respectively. For simplicity, we assume that momentary utility takes the following additively-separable form

$$u(c_t, n_t, m_t) = (1 - \epsilon_t^c) \ln c_t - \frac{\theta}{2} n_t^2 - \frac{\gamma}{2} m_t^2.$$

The term $-\gamma m_t^2/2$ represents the utility cost of using NPIs. Before the epidemic, there is no benefit to using NPIs, so the optimal level of m_t is zero. The variable ϵ_t^c represents the impact of containment measures on the utility from consumption. For example, if the government shuts down bars, one is forced to drink at home alone which, presumably, yields less utility. In pre-epidemic steady state there are no containment measures ($\epsilon_t^c = 0$).

The budget constraint of the representative person is:

$$c_t = w_t n_t, \tag{1}$$

where w_t denotes the real wage rate.

The first-order condition for the representative-person's problem is:

$$\theta n_t = c_t^{-1} w_t.$$

There is a continuum of competitive representative firms of unit measure that produce consumption goods (C_t) using hours worked (N_t) according to the technology:

$$C_t = A N_t.$$

The firm chooses hours worked to maximize its time-t profits Π_t:

$$\Pi_t = A N_t - w_t N_t.$$

In equilibrium, $n_t = N_t$ and $c_t = C_t$.

2.2 The outbreak of an epidemic

As in Kermack and McKendrick (1927), the population consists of four groups: susceptible (people who have not yet been exposed to the virus), infected (people who infected by the virus), recovered (people who survived the infection and acquired immunity), and deceased (people who died from the infection). The fractions of people in these four groups are denoted by S_t, I_t, R_t and D_t, respectively.

In contrast to much of the economic literature on epidemics, we assume that people do not know their true health state unless they are tested.[3]

We denote the state of being alive by a_t. People's time-t subjective probability about whether they are susceptible, infected or recovered is given by $p(s_t|a_t)$, $p(i_t|a_t)$, and $p(r_t|a_t)$, respectively

In every period, a fraction π_r of infected people recover and a fraction π_d die. The timing of events is as follows. Social interactions, including consumption- and work-related activities, happen in the beginning of the period. Then, changes in health states unrelated to social interactions (recovery or death of infected people) occur. Finally, the consequences of social interactions materialize and some susceptible people become infected.

At time zero, there is a fraction ε of the population that is infected:

$$I_0 = \varepsilon,\ S_0 = 1 - \varepsilon.$$

This information is public and it is used by people to form their time-zero health-state subjective probabilities:

$$p(s_0|a_0) = 1 - \varepsilon,\ p(i_0|a_0) = \varepsilon,\ p(r_0|a_0) = 0.$$

People meet in one of three ways: purchasing consumption goods, working, and engaging in non-economic activities. Meetings occur randomly in all social interactions. To simplify, we assume that the probability of a given person being infected through more than one form of social interactions is zero and that that the effect of NPIs on virus transmission is the same for all forms of social interaction.

The representative person's subjective probability of becoming infected by the virus is

$$\tau_t = (1 - m_t)\left[\pi_1 c_t I_t C_t (1 - M_t) + \pi_2 n_t I_t N_t (1 - M_t) + \pi_3 I_t (1 - M_t)\right]. \tag{2}$$

[3] Other models in which people do not know their true health state include those proposed by Brotherhood et al. (2020) and Farboodi et al. (2020).

In equation (2), $I_t C_t$ and $I_t N_t$ are the aggregate consumption and hours worked of infected people. The variable M_t represents the equilibrium level of NPIs adopted in the economy. The terms $\pi_1 c_t \left(I_t C_t\right)\left(1-M_t\right)$ and $\pi_2 n_t \left(I_t N_t\right)\left(1-M_t\right)$ reflect transmissions that result from consumption- and work-related interactions, respectively. The parameter π_1 reflects both the amount of time spent shopping per unit of consumption and the probability that the virus is transmitted as a result of that activity. The parameter π_2 reflects the probability that the virus is transmitted as a result of work interactions. The term $\pi_3 I_t(1-M_t)$ reflects transmissions that result from non-economic interactions.

The representative person's subjective probability of being susceptible at time $t+1$ conditional on being alive at time t is

$$p(s_{t+1}|a_t) = (1-\tau_t)p(s_t|a_t). \tag{3}$$

Here, $(1-\tau_t)$ is the probability of a susceptible person not becoming infected at time t.

The representative person's subjective probability of being infected at time $t+1$, conditional on being alive at time t, is

$$p(i_{t+1}|a_t) = \tau_t p(s_t|a_t) + (1-\pi_r-\pi_d)p(i_t|a_t). \tag{4}$$

Here, $\tau_t p(s_t|a_t)$ is the subjective probability of being susceptible at time t and becoming infected at time $t+1$. The term $(1-\pi_r-\pi_d)p(i_t|a_t)$ is the subjective probability that a person infected at time t survives until time $t+1$ but does not recover from the infection.

The representative person's subjective probability of being recovered at time $t+1$, conditional on being alive at time t is

$$p(r_{t+1}|a_t) = p(r_t|a_t) + \pi_r p(i_t|a_t). \tag{5}$$

Here, $p(r_t|a_t)$ is the probability that the person has already recovered at time t. In that case, the person remains recovered at time $t+1$. The term $\pi_r p(i_t|a_t)$ is the probability of being infected at time t and recovered at time $t+1$.

Using the following conditions

$$p(s_{t+1}|a_{t+1}) = \frac{p(s_{t+1}|a_t)}{1-\pi_d p(i_t|a_t)},$$

$$p(i_{t+1}|a_{t+1}) = \frac{p(i_{t+1}|a_t)}{1-\pi_d p(i_t|a_t)},$$

$$p(r_{t+1}|a_{t+1}) = \frac{p(r_{t+1}|a_t)}{1-\pi_d p(i_t|a_t)},$$

we can rewrite equations (4), (3), and (5) as

$$p(i_{t+1}|a_{t+1})\left[1-\pi_d p(i_t|a_t)\right]=\tau_t p(s_t|a_t)+(1-\pi_r-\pi_d)p(i_t|a_t), \tag{6}$$

$$p(s_{t+1}|a_{t+1})\left[1-\pi_d p(i_t|a_t)\right]=p(s_t|a_t)(1-\tau_t), \tag{7}$$

$$p(r_{t+1}|a_{t+1})\left[1-\pi_d p(i_t|a_t)\right]=p(r_t|a_t)+\pi_r p(i_t|a_t). \tag{8}$$

2.3 The problem of the representative person

Since everyone has the same subjective probabilities about their health state, everybody chooses the same level of consumption (c_t) and hours worked (n_t). The lifetime utility of the representative person at time t, U_t, is given by

$$U_t=\sum_{j=0}^{\infty}\beta^j p(a_{t+j}|a_t)u\left(c_{t+j},n_{t+j},m_{t+j}\right),$$

where $p(a_{t+j}|a_t)$ is the probability of a person being alive at time $t+j$ given that the person is alive at time t.

We can write U_t in recursive form as

$$U_t=u(c_t,n_t,m_t)+\beta\left[1-\pi_d p(i_t|a_t)\right]U_{t+1}. \tag{9}$$

To derive the first-order conditions, it is useful to write

$$U_{t+1}=u(c_{t+1},n_{t+1},m_{t+1})+\beta\left[1-\pi_d p(i_{t+1}|a_{t+1})\right]U_{t+2},$$

and note the fact that

$$\begin{aligned}\frac{dU_t}{dp(i_{t+1}|a_{t+1})} &= \beta\left[1-\pi_d p(i_t|a_t)\right]\frac{dU_{t+1}}{p(i_{t+1}|a_{t+1})},\\ &= -\beta^2\left[1-\pi_d p(i_t|a_t)\right]\pi_d U_{t+2}.\end{aligned}$$

The problem of the representative person is to maximize (9) subject to the budget constraint, (1), the transmission function, (2), and the probability equations (6) and (7).[4]

The first-order conditions with respect to c_t, n_t, m_t, τ_t, $p(i_{t+1}|a_{t+1})$, and $p(s_{t+1}|a_{t+1})$ are given by

$$(1-\epsilon_t^c)\frac{1}{c_t}-\lambda_t^b+\lambda_t^\tau(1-m_t)\pi_1 I_t C_t(1-M_t)=0,$$

[4]Equation (8) is redundant since $p(s_{t+1}|a_{t+1})+p(i_{t+1}|a_{t+1})+p(r_{t+1}|a_{t+1})=1$.

$$-\theta n_t + \lambda_t^b A + \lambda_t^\tau (1-m_t)\pi_2 I_t N_t (1-M_t) = 0,$$

$$-\gamma m_t = \lambda_t^\tau \frac{\tau_t}{1-m_t},$$

$$-\lambda_t^\tau + \lambda_t^i p(s_t|a_t) - \lambda_t^s p(s_t|a_t) = 0,$$

$$-\beta^2 \pi_d U_{t+2} - \lambda_t^i + \beta \lambda_{t+1}^i [1 - \pi_r - \pi_d (1 - p(i_{t+2}|a_{t+2}))] + \beta \lambda_{t+1}^s \pi_d p(s_{t+2}|a_{t+2}) = 0,$$

$$\beta \lambda_{t+1}^i \tau_{t+1} - \lambda_t^s + \beta \lambda_{t+1}^s (1-\tau_{t+1}) = 0.$$

Here, $\lambda_{t+j}^b \beta^j p(a_{t+j}|a_t)$, $\lambda_{t+j}^\tau \beta^j p(a_{t+j}|a_t)$, $\lambda_{t+j}^i \beta^j p(a_{t+j}|a_t)$, and $\lambda_{t+j}^s \beta^j p(a_{t+j}|a_t)$ denote the Lagrange multipliers associated with constraints (1), (2), (6), and (7), respectively.

Equilibrium In equilibrium, each person solves their maximization problem. In addition, the goods and labor markets clear:

$$(S_t + I_t + R_t)\, c_t = A N_t,$$

$$(S_t + I_t + R_t)\, n_t = N_t.$$

Given rational expectations, the subjective and objective probabilities of different health states coincide:

$$\begin{aligned} S_t &= p(s_t|a_0), \\ I_t &= p(i_t|a_0), \\ R_t &= p(a_t|a_0) - p(s_t|a_0) - p(i_t|a_0), \\ D_t &= 1 - p(a_t|a_0). \end{aligned}$$

where

$$\begin{aligned} p(s_t|a_0) &= p(s_t|a_t)p(a_t|a_{t-1})p(a_{t-1}|a_{t-2})...p(a_1|a_0), \\ p(i_t|a_0) &= p(i_t|a_t)p(a_t|a_{t-1})p(a_{t-1}|a_{t-2})...p(a_1|a_0), \\ p(a_t|a_0) &= p(a_t|a_{t-1})p(a_{t-1}|a_{t-2})...p(a_1|a_0), \end{aligned}$$

and

$$p(a_t|a_{t-1}) = 1 - \pi_d p(i_{t-1}|a_{t-1}).$$

3 Model with testing and quarantining

In this section, we discuss the impact of testing and quarantining. To highlight the key mechanisms through which this policy affects the economy, we assume that tests perfectly reveal people's health state. In reality, tests have both type one and two errors.[5] Allowing for these errors would greatly complicate the analysis without changing the basic insights. With imperfect testing, people would use all their prior test results to estimate their current health status. Absent some simplification, the number of different types of people would grow without bound.

To capture the idea that it takes time to build testing capacity, we proceed as follows. The population is divided into a testing pool and a non-testing pool. Those in the testing pool are tested every period until they recover or die. Initially, the government tests α percent of the population. In each subsequent period, the government adds another α percent of the population to the testing pool. The number of people in the testing pool grows over time in a way that depends on α.

The testing pool is a convenient way to minimize the heterogeneity induced by testing. Without the testing pool, even with perfect testing, we would have to keep track of a growing number of types of people defined by the time of their last test and the test result.

Taken together, our simplifying assumptions bound the degree of heterogeneity in the economy because the timing of entry into the testing pool does not affect current consumption or work decisions. All that matters for these decisions is a person's current health state.

The government's quarantine policy is as follows. People who test positive for infection are not allowed to work or go shopping but receive consumption goods from the government in a way that bypasses social interactions. For convenience, we assume that infected people in the testing pool are required to adopt the same level of NPI use as susceptible people in the testing pool.

We now discuss the maximization problem of people inside and outside the testing pool. We use the superscripts u and k to denote variables that pertain to people with unknown and known health states, respectively.

[5] See Berger et al. (2021) for a discussion of different types of tests and their trade-offs, as well as an analysis of testing and quarantining in a non-behavioral SIR model.

3.1 People outside the testing pool

People outside the testing pool are uncertain about their current health state. Those who survive, enter the testing pool at time $t+1$ with probability α and will, from then on, know their health state at each point in time.

We assume that testing starts in period 0, so the initial conditions for the different groups in the population are:

$$I_0^u = \varepsilon,\ S_0^u = 1-\varepsilon, \text{ and } S_0^k = I_0^k = R_0^u = R_0^k = 0.$$

The probability that a given person outside the testing pool is susceptible, infected or recovered at time zero is given by

$$p(s_0|a_0) = 1-\varepsilon,\ p(i_0|a_0) = \varepsilon,\ p(r_0|a_0) = 0,$$

respectively.

The lifetime utility of a person who is outside the testing pool, U_t^u, is given by

$$\begin{aligned} U_t^u &= u(c_t^u, n_t^u, m_t^u) + (1-\alpha)\beta\left[1-\pi_d p(i_t|a_t)\right] U_{t+1}^u \\ &\quad +\alpha\beta\left[1-\pi_d p(i_t|a_t)\right]\left[p(s_{t+1}|a_{t+1})U_{t+1}^s + p(i_{t+1}|a_{t+1})U_{t+1}^i + p(r_{t+1}|a_{t+1})U_{t+1}^r\right]. \end{aligned} \tag{10}$$

The variables U_{t+1}^s, U_{t+1}^i, and U_{t+1}^r denote the lifetime utility of a person who is susceptible, infected, and recovered at time $t+1$, respectively.

In deriving the first-order conditions of a person's maximization problem, it is useful to write U_{t+1}^u as

$$\begin{aligned} U_{t+1}^u &= u(c_{t+1}^u, n_{t+1}^u, m_{t+1}^u) + (1-\alpha)\beta\left[1-\pi_d p(i_{t+1}|a_{t+1})\right] U_{t+2}^u \\ &\quad \alpha\beta\left[1-\pi_d p(i_{t+1}|a_{t+1})\right]\left[p(s_{t+2}|a_{t+2})U_{t+2}^s + p(i_{t+2}|a_{t+2})U_{t+2}^i + p(r_{t+2}|a_{t+2})U_{t+2}^r\right]. \end{aligned}$$

The problem of a person outside the testing pool is to maximize (10) subject to the budget constraint, the transmission function, and the laws of motion for the probability of being infected and susceptible.

The budget constraint is given by

$$c_t^u = An_t^u + \Gamma_t^u, \tag{11}$$

where Γ_t^u denotes a lump-sum transfer from the government.

The transmission function is given by

$$\begin{aligned}\tau_t^u &= (1-m_t^u)\{\pi_1 c_t^u[I_t^u C_t^u(1-M_t^u)+\chi_t I_t^k C_t^i(1-M_t^i)] \\ &+\pi_2 n_t^u[I_t^u N_t^u(1-M_t^u)+\chi_t I_t^k N_t^i(1-M_t^i)] \\ &+\pi_3[I_t^u(1-M_t^u)+I_t^k(1-M_t^i)]\},\end{aligned} \tag{12}$$

where χ_t is an indicator function that takes the value one if people who test positive for infection are quarantined and zero otherwise. Recall that people who test positive do not work or shop but receive consumption goods from the government in a way that bypasses social interactions.

The laws of motion for the probability of being infected and susceptible are

$$p(i_{t+1}|a_{t+1})[1-\pi_d p(i_t|a_t)] = \tau_t^u p(s_t|a_t) + (1-\pi_r-\pi_d)p(i_t|a_t), \tag{13}$$

$$p(s_{t+1}|a_{t+1})[1-\pi_d p(i_t|a_t)] = p(s_t|a_t)(1-\tau_t^u), \tag{14}$$

where m_t^u is the level of NPIs adopted by people who do not know their health status.

The first-order conditions with respect to c_t^u, n_t^u, m_t^u, τ_t^u, $p(i_{t+1}|a_{t+1})$, and $p(s_{t+1}|a_{t+1})$ are given by

$$(1-\epsilon_t^c)\frac{1}{c_t^u} - \lambda_{bt}^u + \lambda_{\tau t}^u(1-m_t^u)\pi_1[I_t^u C_t^u(1-M_t^u)+\chi_t I_t^k C_t^i(1-M_t^i)] = 0,$$

$$-\theta n_t^u + \lambda_{bt}^u A + \lambda_{\tau t}^u(1-m_t^u)\pi_2[I_t^u N_t^u(1-M_t^u)+\chi_t I_t^k N_t^i(1-M_t^i)] = 0,$$

$$-\gamma m_t^u = \lambda_{\tau t}^u \frac{\tau_t^u}{1-m_t^u}$$

$$-\lambda_{\tau t}^u + \lambda_{it}^u p(s_t|a_t) - \lambda_{st}^u p(s_t|a_t) = 0,$$

$$\begin{aligned}&\frac{dU_t^u/dp(i_{t+1}|a_{t+1})}{1-\pi_d p(i_t|a_t)} - \lambda_{it}^u + \beta\lambda_{it+1}^u \pi_d p(i_{t+2}|a_{t+2}) + \\ &\beta\lambda_{it+1}^u(1-\pi_r-\pi_d) + \beta\lambda_{st+1}^u \pi_d p(s_{t+2}|a_{t+2}) = 0,\end{aligned}$$

$$\frac{dU_t^u/dp(s_{t+1}|a_{t+1})}{1-\pi_d p(i_t|a_t)} + \beta\lambda_{it+1}^u \tau_{t+1}^u - \lambda_{st}^u + \beta\lambda_{st+1}^u(1-\tau_{t+1}^u) = 0.$$

Here, $\lambda_{bt+j}^u \beta^j p(a_{t+j}|a_t)$, $\lambda_{\tau t+j}^u \beta^j p(a_{t+j}|a_t)$, $\lambda_{it+j}^u \beta^j p(a_{t+j}|a_t)$, and $\lambda_{st+j}^u \beta^j p(a_{t+j}|a_t)$ denote the Lagrange multipliers associated with constraints (11), (12), (13), and (14), respectively.

The aggregate distribution of people outside the testing pool, according to health states is given by

$$\begin{aligned} S^u_{t+1} &= p(s_{t+1}|a_0)(1-\alpha)^t, \\ I^u_{t+1} &= p(i_{t+1}|a_0)(1-\alpha)^t, \\ R^u_{t+1} &= [p(a_{t+1}|a_0) - p(s_{t+1}|a_0) - p(i_{t+1}|a_0)](1-\alpha)^t. \end{aligned}$$

3.2 People inside the testing pool

People inside the testing pool know whether they are susceptible, infected or recovered at time t. People who are susceptible and infected face uncertainty about their future health state. The indexes s, i, and r, denote infected, susceptible and recovered people, respectively.

A person of type $j \in \{s, r\}$ has the budget constraint

$$c^j_t = w_t n^j_t + \Gamma^j_t, \tag{15}$$

where Γ^j_t is a lump sum transfer from the government.

People who are infected are quarantined. They do not work ($n^i_t = 0$) and their consumption is provided by the government in a way that bypasses social interactions. We assume that the government provides infected people the same level of consumption as recovered people.

$$c^i_t = c^r_t.$$

We now describe the optimization problem of the different people inside the testing pool.

Susceptible people The lifetime utility of a susceptible person, U^s_t, is

$$U^s_t = u(c^s_t, n^s_t, m^s_t) + \beta\left[(1-\tau^s_t)\, U^s_{t+1} + \tau^s_t U^i_{t+1}\right], \tag{16}$$

where τ^s_t represents the probability that a susceptible person becomes infected. Susceptible people understand that consuming and working less reduces this probability, which is given by the following transmission function

$$\begin{aligned} \tau^s_t &= (1-m^s_t)\{\pi_1 c^s_t[I^u_t C^u_t(1-M^u_t) + \chi_t I^k_t C^i_t(1-M^i_t)] \\ &\quad +\pi_2 n^s_t[I^u_t N^u_t(1-M^u_t) + \chi_t I^k_t N^i_t(1-M^i_t)] \\ &\quad +\pi_3[I^u_t(1-M^u_t) + I^k_t(1-M^i_t)]\} \end{aligned} \tag{17}$$

where m_t^s is the level of NPIs chosen by susceptible people. The variables M_t^u and M_t^i are the equilibrium levels of NPIs for people who do not know their health status and people who know they are infected, respectively. Recall that people who tested positive for infections are not allowed to work or go shopping but receive consumption goods from the government in a way that bypasses social interactions. So, their consumption and labor supply do not enter the transmission function.

The first-order conditions for consumption, hours worked and NPI use are

$$(1-\epsilon_t^c)\frac{1}{c_t^s} - \lambda_{bt}^s + (1-m_t^s)\lambda_{\tau t}^s \pi_1 [I_t^u C_t^u (1-M_t^u) + \chi_t I_t^k C_t^i (1-M_t^i)] = 0,$$

$$-\theta n_t^s + A\lambda_{bt}^s + (1-m_t^s)\lambda_{\tau t}^s \pi_2 [I_t^u N_t^u (1-M_t^u) + \chi_t I_t^k N_t^i (1-M_t^i)] = 0,$$

$$-\gamma m_t^s = \lambda_{\tau t}^s \frac{\tau_t^s}{1-m_t^s}.$$

Here, λ_{bt}^s and $\lambda_{\tau t}^s$ are the Lagrange multipliers associated with constraints (15) and (17), respectively.

The first-order condition for τ_t^s is

$$\beta \left(U_{t+1}^i - U_{t+1}^s\right) - \lambda_{\tau t}^s = 0. \tag{18}$$

Infected people The lifetime utility of an infected person, U_t^i, is

$$U_t^i = u(c_t^i, n_t^i, m_t^i) + \beta \left[(1-\pi_r-\pi_d)\, U_{t+1}^i + \pi_r U_{t+1}^r\right]. \tag{19}$$

The expression for U_t^i embodies a common assumption in macro and health economics that the cost of death is the foregone utility of life.

In the absence of quarantines, the first-order conditions for consumption and hours worked are given by

$$u_1(c_t^i, n_t^i, m_t^i) = \lambda_{bt}^i,$$

$$u_2(c_t^i, n_t^i, m_t^i) = -A\lambda_{bt}^i,$$

where λ_{bt}^i is the Lagrange multiplier associated with constraint (15).[6]

The first-order conditions for NPI use is

$$-\gamma m_t^i \leq 0.$$

[6] We assume that infected people are as productive as other people. Absent this assumption people could learn whether they are infected based on their productivity.

In the presence of quarantines, consumption, hours worked and NPI use are given by $c_t^i = c_t^r$, $n_t^i = 0$, and $m_t^i = m_t^s$.

Recovered people The lifetime utility of a recovered person, U_t^r, is

$$U_t^r = u(c_t^r, n_t^r, m_t^r) + \beta U_{t+1}^r. \tag{20}$$

The first-order conditions for consumption, hours worked and NPI use are

$$(1 - \epsilon_t^c) \frac{1}{c_t^r} = \lambda_{bt}^r,$$

$$-\theta n_t^r = -A\lambda_{bt}^r,$$

$$-\gamma m_t^r \leq 0,$$

where λ_{bt}^r is the Lagrange multiplier associated with constraint (15).

Equilibrium In equilibrium, group-specific aggregates and individual levels of consumption, hours worked and NPI use coincide:

$$c_t^j = C_t^j,\ n_t^j = N_t^j, \text{ and } m_t^j = M_t^j,$$

where $j \in \{s, i, r, u\}$.

The government budget constraint holds:

$$\Gamma_t \left(S_t^k + R_t^k + S_t^u + I_t^u + R_t^u\right) + \Gamma_t^i I_t^k = 0,$$

where Γ_t^i is a positive lump-sum transfer that finances the consumption of the infected and quarantined. The variable $\Gamma_t = \Gamma_t^j$ for $j = s, r, u$ is a negative lump-sum transfer on everybody else.

In equilibrium, each person solves their maximization problem and the government budget constraint is satisfied. In addition, the goods and labor markets clear:

$$\left(S_t^k C_t^s + I_t^k C_t^i + R_t^k C_t^r\right) + \left(S_t^u + I_t^u + R_t^u\right) C_t^u = AN_t,$$

$$\left(S_t^k N_t^s + R_t^k N_t^r\right) + \left(S_t^u + I_t^u + R_t^u\right) N_t^u = N_t.$$

Population dynamics We now describe how the size of different groups in the economy evolves over time. The aggregate number of new infections among people outside the testing pool (T_t^u) is equal to the number of viral transmissions (τ_t^u, defined in equation (12)) times the fraction of people outside the testing pool that survived from period zero to period t and are susceptible ($p(s_t|a_0)$)

$$T_t^u = \tau_t^u p(s_t|a_0).$$

The aggregate number of new infections among people inside the testing pool (T_t^k) is equal to:

$$\begin{aligned} T_t^k &= \pi_1 S_t^k C_t^s (1 - M_t^s)[I_t^u C_t^u (1 - M_t^u) + \chi_t I_t^k C_t^i (1 - M_t^i)] + \\ &\quad \pi_2 S_t^k (1 - M_t^s) N_t^s [I_t^u N_t^u (1 - M_t^u) + \chi_t I_t^k N_t^i (1 - M_t^i)] \\ &\quad + \pi_3 (1 - M_t^s) S_t^k [I_t^u (1 - M_t^u) + I_t^k (1 - M_t^i)]. \end{aligned} \tag{21}$$

This equation is an aggregate, equilibrium version of equation (17) that takes into account that there are S_t^k susceptible people in the testing pool.

Recall that social interactions which occur at time t lead to changes in the health state of susceptible people at the end of time t. So, the number of susceptible people at the end of period t inside and outside of the testing pool is $S_t^k - T_t^k$ and $S_t^u - T_t^u$, respectively.

The number of susceptible people in the testing pool at time $t+1$ is equal to the number of susceptible people in the testing pool at the end of time t ($S_t^k - T_t^k$), plus the number of people outside the testing pool who got tested for the first time in the beginning of period $t+1$ and learned they are susceptible ($\alpha(S_t^u - T_t^u)$):

$$S_{t+1}^k = S_t^k - T_t^k + \alpha(S_t^u - T_t^u). \tag{22}$$

The number of susceptible people outside the testing pool at the beginning of $t+1$ is equal to the number of susceptible people who were outside of the pool at the end of period t and did not get tested in the beginning of time $t+1$:

$$S_{t+1}^u = (1 - \alpha)(S_t^u - T_t^u). \tag{23}$$

The number of infected people in the testing pool at the beginning of time $t+1$ is equal to the number of newly infected people (T_t^k) in the testing pool, plus the number of infected people in the testing pool at the beginning of time t (I_t^k), minus the number of infected people in the testing pool who either recovered ($\pi_r I_t^k$) or died ($\pi_d I_t^k$), plus the number of

people outside the testing pool who got tested for the first time at the beginning of time $t+1$ and learned that they are infected ($\alpha\left[T_t^u + (1-\pi_r-\pi_d)\, I_t^u\right]$):

$$I_{t+1}^k = T_t^k + (1-\pi_r-\pi_d)\, I_t^k + \alpha\left[T_t^u + (1-\pi_r-\pi_d)\, I_t^u\right].$$

The number of infected people outside the testing pool at the beginning of time $t+1$ is equal to the number of infected people who were outside of the pool at the end of time t ($T_t^u + (1-\pi_r-\pi_d)\, I_t^u$) and did not get tested at the beginning of time $t+1$:

$$I_{t+1}^u = (1-\alpha)[T_t^u + (1-\pi_r-\pi_d)\, I_t^u].$$

The number of recovered people in the testing pool at time $t+1$ is the number of recovered people in the testing pool at beginning of time t (R_t^k), plus the number of infected people in the testing pool who just recovered ($\pi_r I_t^k$), plus the number of people outside the testing pool who got tested for the first time at the beginning of period $t+1$ and learned they are recovered ($\alpha\left(R_t^u + \pi_r I_t^u\right)$):

$$R_{t+1}^k = R_t^k + \pi_r I_t^k + \alpha\left(R_t^u + \pi_r I_t^u\right). \tag{24}$$

The number of recovered people outside the testing pool at the beginning of time $t+1$ is equal to the number of recovered people who were outside the pool at the end of time t and did not get tested at the beginning of time $t+1$:

$$R_{t+1}^u = (1-\alpha)(R_t^u + \pi_r I_t^u). \tag{25}$$

Finally, the number of deceased people at time $t+1$ is the number of deceased people at time t plus the number of new deaths ($\pi_d\left(I_t^u + I_t^k\right)$):

$$D_{t+1} = D_t + \pi_d\left(I_t^u + I_t^k\right).$$

The number of tests administered at time t is given by

$$\begin{aligned} \text{Test}_t &= S_t^k + I_t^k + \alpha(S_t^u - T_t^u) + \alpha\left[T_t^u + (1-\pi_r-\pi_d)\, I_t^u\right] + \alpha\left(R_t^u + \pi_r I_t^u\right) \\ &= S_t^k + I_t^k + \alpha\left[S_t^u + (1-\pi_d)\, I_t^u + R_t^u\right]. \end{aligned}$$

To interpret this equation, recall that we test all the people in the testing pool who are not recovered or dead. In addition, we test a fraction α of the people outside the testing pool.

4 Quantitative results

In this section, we discuss our choice of parameter values and quantitative results.

4.1 Parameter values

In the model, each time period represents a week, with time zero corresponding to the first week of February, 2020.

We assume that it takes on average two weeks for Covid infections to resolve, a value that is in the range of the estimates reported by the CDC.[7] Since our model is weekly, we set $\pi_r + \pi_d = 7/14$. As it turns out, our results are relatively insensitive to this choice.

A statistic widely used to diagnose the severity of an epidemic is the "basic reproduction number," $\mathcal{R}_0$. This statistic is the total number of infections caused by one infected person (with measure zero) in his or her lifetime in a population in which everybody is susceptible ($S_0 = 1$). The higher is the value of $\mathcal{R}_0$, the faster is the spread of the virus. There is considerable uncertainty about the values of variables like $\mathcal{R}_0$ and parameters such as the case-fatality rate, π_d. The Center for Disease Control (CDC) provides a range of estimates combined into five scenarios that are broadly consistent with the data.[8] In these scenarios, $\mathcal{R}_0$ ranges from 2 to 4 and the average case-fatality ratio ranges from 0.4 to 1.3 percent.[9] We have diffuse priors over these scenarios, so we chose the scenario that allows our model to provide the best fit to the time series for U.S. Covid deaths.

We calibrate the parameter that controls the utility cost of NPIs, γ, the initial seed of the epidemic, ε, the case-fatality rate, π_d, and the parameters of the transmission function, π_1, π_2, and π_3, as follows. Denote by x the fraction of total infections at the beginning of the epidemic that is accounted for by non-economic interactions, $x = \pi_3 I_0 S_0 / T_0$. We assume that consumption and working activities each account for $(1-x)/2$ percent of total infections at the beginning of the epidemic. We choose γ, x, π_d and ε so that $\mathcal{R}_0 = 2$ and the model comes as close as possible to fitting three moments of the time series for U.S. Covid deaths. The three moments that we target are the number of Covid deaths in February and November, 2020 as well as the average slope of the cumulative death function between these

[7] See https://www.cdc.gov/coronavirus/2019-ncov/hcp/duration-isolation.html and https://www.cdc.gov/coronavirus/2019-ncov/symptoms-testing/symptoms.html#:~:text=Symptoms%20may%20appear%202%2D,exposure%20to%20the%20virus

[8] https://www.cdc.gov/coronavirus/2019-ncov/hcp/planning-scenarios.html

[9] The average case-fatality rates are computed using the demographic weights for the U.S. population.

two points. The resulting parameter values are reported in Table 1.

Table 1: Selected parameter values

Parameter	Value	Description
Calibrated parameters to obtain $\mathcal{R}_0 = 2$ and three moments of Covid deaths data		
γ	1	Disutility of NPI use
x	1/6	Share of initial infections due to general infections
π_d	$7 \times 0.0035/14$	Case-fatality rate (weekly)
ε	0.00066	Initial infection
Implied transmission function parameters		
π_1	3.3584×10^{-7}	Coefficient for consumption-based infections
π_2	5.3293×10^{-4}	Coefficient for labor-based infections
π_3	0.1671	Coefficient for general infections

The initial population is normalized to one. We choose $A = 39.835$ and $\theta = 0.001275$ so that in the pre-epidemic steady state the representative person works 28 hours per week and earns a weekly income of $\$58,000/52$. We obtain the per-capita income for 2019 from the U.S. Bureau of Economic Analysis and the average number of hours worked from the Bureau of Labor Statistics 2018 time-use survey. We set $\beta = 0.96^{1/52}$ so that the value of a life is 9.3 million 2019 U.S. dollars in the pre-epidemic steady state. This value is consistent with the economic value of life used by U.S. government agencies in their decisions process.[10]

To calibrate the model, we must take a stand on the containment measures introduced by the government. In practice, there was substantial heterogeneity across state and local government both with respect to timing and the precise nature of the measures adopted.

In our model, the variable ϵ_t^c summarizes these government containment measures. We choose the level and time path for ϵ_t^c so that the model is consistent with the behavior of consumption at a monthly frequency in the period from March to May. We set ϵ_t^c equal to 0.1 and 0.35 in March and April, respectively. The value of ϵ_t^c is zero in all other periods. The time path for ϵ_t^c is broadly consistent with the evolution of mandatory-for-all stay-at-home orders documented by Moreland et al. (2020).

Figure 1 shows the data and model-implied paths for monthly aggregate consumption and weekly cumulative Covid deaths in the first year of the epidemic. The black line corresponds to the data up to November 2020. The pink line corresponds to the model with containment.

[10] See U.S. Environmental Protection Agency (2010) and Moran (2016). See Viscusi and Aldy (2003) for a review of the literature on the value of a statistical life.

According to Figure 1, our model accounts for the broad rise and total number of Covid deaths.[11] By construction, the model captures well the dynamics of consumption between February and May, 2020. The model somewhat understates the recovery of consumption in the summer and fall. This understatement could reflect three factors. The first factor is seasonality in rates of infection (see e.g. Merow and Urban (2020)). The second factor is that businesses reorganized to reduce the probability that workers and customers get infected. These effects could be incorporated into the model, but they would add complexity to the analysis without altering the key mechanisms discussed below. The third factor is that people substituted towards forms of consumption and work that reduce the probability of getting infected. Examples include e-commerce and remote work.[12]

To isolate the effect of various mechanisms in our model, we display in Figure 1 the counterfactual paths for consumption and total Covid deaths for an economy without containment measures ($\epsilon_t^c = 0$ for all t). From the perspective of our model, containment had a large impact on consumption but a relatively small impact on cumulative aggregate deaths.

We consider a range of values for α. The U.S. did relatively few random tests, so we assume that $\alpha = 0$ in the benchmark model.[13] We then consider three other values of α: 0.25 percent, 0.5 percent, and 2 percent. To interpret these values it is useful to compute the number of tests per person administered in the first six months and in the first year of the epidemic. Table 2 reports these values as well as the percentage of the population in the

[11] For a detailed analysis of the dynamics of Covid-related deaths across many countries see Atkeson, Kopecky, and Zha (2020).

[12] See Jones, Philippon, and Venkateswaran (2020) and Krueger, Uhlig, and Xie (2020) for analyses of these forms of substitution.

[13] According to Hasell et al. (2020), between March 2020 and March 2021 the U.S. administered about one test per person. Most of these tests were not randomly administered as part of a testing and quarantining policy. Instead, tests were used to diagnose people who exhibited Covid symptoms.

testing pool at the end of the first six months and the first year.

Table 2: Tests in model economy

	Number of tests per person	
α	End of first 6 months	End of first year
0.0025	0.8	2.7
0.005	1.6	5.3
0.02	5.7	18.2
	Percentage of population in testing pool	
	End of first 6 months	End of first year
0.0025	5	8
0.005	10	17
0.02	37	57

To put these numbers into perspective, it is useful to consider the testing policies implemented in major U.S. universities. By the fall of 2020, 20 percent of U.S. universities began regular testing programs, implementing the type of large-scale testing advocated by Romer (2020). According to Booeshaghi et al. (2020), the modal number of weekly tests per student among universities with testing programs is two. Few tests were administered between March and September, 2020. So, on average, students were tested roughly once a week between March 2020 to March 2021, resulting in 52 tests per student during the first year of the epidemic. From this perspective, the number of tests implied by the values of α that we consider were technically feasible.

4.2 The effect of NPIs in isolation

To assess the effect of NPIs, we abstract from testing and quarantining as well as from containment ($\epsilon_t^c = 0$ for all t). We compare the baseline model ($\gamma = 1$) with economies that have a low ($\gamma = 0.3$) and a high ($\gamma = 3$) disutility of NPIs, respectively. We refer to these models as the baseline, low- and high-γ economies, respectively.

Figure 2 displays how key aggregate variables evolve over time for the three values of γ. The (1,1) element of Figure 2 displays the number of infected people as a percentage of the initial population. Two key properties are worth noting. First, the peak number of infected people is sharply increasing in γ. For example, in the baseline economy the peak is equal to 2.1 percent of the initial population. The peaks in the low- and high-γ economy are 0.8 percent and 4 percent, respectively. Second, infections converge to zero more slowly when γ is lower. When $\gamma = 3$, the number of infections is roughly zero after two years. When

$\gamma = 0.3$, infections do not reach values close to zero six years into the epidemic. So, on net, NPIs reduce the intensity of the epidemic but prolong its duration.

The (2,3) element of Figure 2 shows the impact of γ on NPI use. The overall dynamics of NPI use are similar for different values of γ. However, there are two important differences. First, NPI use declines with γ. Second, NPI use converges to zero more slowly when γ is lower, consistent with the longer duration of the epidemic.

The (2,1) element of Figure 2 displays the dynamics of cumulative deaths. Consistent with the impact of γ on infections, most of the deaths in the high-γ economy occur in the first year of the epidemic. Thereafter, the number of new deaths is small. In contrast, there are relatively few deaths in the low-γ economy during the first year. But, there are a large number of deaths thereafter.

To further explore the impact of γ on total Covid deaths, we turn to Figure 3. The blue line in the (1,1) element of that figure displays total epidemic-related deaths as a function of γ. The key property of this line is its flatness. Low values of γ promote more intense use of NPIs. That intensity delays deaths from the epidemic. Critically, it does not substantially reduce the overall death toll.

We now turn to the economic impact of the epidemic. The (2,2) element of Figure 2 shows the path for consumption. The qualitative behavior of consumption mirrors the path of infections. The smaller is γ, the lower is the peak-to-trough decline in consumption. However, low values of γ are associated with more persistent recessions.

The blue line in element (1,2) of Figure 3 shows the impact of γ on the cumulative drop in consumption. We measure the latter as the sum, over the first ten years, of the difference between the equilibrium consumption path during the epidemic and the pre-epidemic level of consumption. Like the analogue line for epidemic-related deaths, the key property of this line is its flatness. Low values of γ delay the drop in consumption but do not substantially reduce its cumulative decline.

Taken together, our results imply that the main effect of NPIs is to buy time for PIs to mitigate the overall impact of the epidemic on people's health and the economy.

4.3 A model with NPIs and testing/quarantining

To isolate the effect of testing and quarantining, we abstract from containment ($\epsilon_t^c = 0$ for all t). Figure 4 displays our results for $\alpha = 0.005$. The blue line corresponds to the competitive equilibrium without testing. The red line corresponds to the equilibrium with testing and

quarantining. Two key results emerge from Figure 4. First, testing and quarantining cuts peak infection rates from 2.1 percent to 1.5 percent and reduces total death rates from 0.2 percent to 0.1 percent of the initial population. Second, testing and quarantining reduces the severity of the recession induced by the epidemic. In the equilibrium with $\alpha = 0$, the peak-to-trough drop in consumption is 6.9 percent. With testing and quarantining, the peak-to-trough drop in consumption falls to 6.5 percent. So, testing and quarantining improves both health and economic outcomes.

When containment measures don't condition on people's health status, there is an extremely painful trade-off between the severity of a recession and the health consequences of an epidemic (see, for example, Alvarez, Argente, and Lippi (2020), Boppart et al. (2020), and Eichenbaum, Rebelo and Trabandt (2020)). Our results show that testing and quarantining dramatically improve this trade-off.

To understand the mechanisms underlying the impact of testing and quarantining, Figure 5 displays consumption and hours worked by different types of people. The first and second rows correspond to the competitive equilibrium and the economy with testing and quarantining, respectively.

Our key results are as follows. The consumption of people who know they are susceptible because of testing drops by more than the consumption of people who do not know their health state. The reason is that they know with certainty that they are at risk of being infected and so they are more cautions in their behavior. The consumption of people who know they are recovered is, up to the effects of lump-sum taxes, the same as in the pre-epidemic steady state. Because of government policy the consumption of infected and recovered people is, up to lump-sum taxes, the same.

The economy with testing and quarantining reaches herd immunity after two years. This immunity is attained for two reasons. First, because testing ramps up gradually, many infected people who are not quarantined continue to spread the virus during the first year. Second, during the same time period infected people who are quarantined continue to transmit the virus through non-economic social interactions. Both forces reduce the pool of susceptible people to the point where herd immunity is obtained.

We now discuss how the gains from testing and quarantining depend on the fraction of the population added every week to the testing pool. Figure 6 displays, for various values of α, the cumulative fall in consumption, the death toll from the epidemic, as well as the peak infection rate. As α rises, both the economic and health costs of the epidemic decline. The

economic cost declines quite steeply as α rises from zero. A rise in α from zero to 2 percent cuts the cumulative fall in consumption from 500 percent to 200 percent.

Further rises in α continue to reduce the economic cost of the epidemic but at a slower rate, with very small reductions beyond $\alpha = 0.06$. A similar, but less stark pattern emerges regarding the death toll from the epidemic. For example, a rise in α from zero to 2 percent cuts the death toll as a percentage of the initial population from 0.18 to 0.06 percent of the initial population. Further rises in α continue to reduce the death toll but at a slower rate.

4.4 The synergy between NPIs and testing/quarantining

We now discuss the synergy between NPIs and testing/quarantining on health outcomes. Figure 3 displays total deaths as a function of γ. The blue line corresponds to the equilibrium without testing and quarantining. The dashed-black, pink and green lines correspond to the equilibrium with α equal to 0.0025, 0.005 and 0.02, respectively.

One way to see the synergies is to focus, for a given γ, on the *difference* between the blue and different dashed lines. That difference corresponds to the impact of testing and quarantining on terminal deaths and the cumulative drop in consumption. To be concrete, consider the case of $\alpha = 0.0025$. The impact of testing and quarantining on lives saved gets dramatically larger as γ gets smaller, i.e. as there is more intensive use of NPIs. When $\gamma = 3$, testing and quarantining reduces total deaths as a percentage of the initial population from 0.20 percent to 0.17 percent. When $\gamma = 0.3$, total deaths fall from 0.18 percent to 0.1 percent. So, the impact of testing and quarantining is much larger when γ is smaller.

The source of this synergy is that NPIs buy time for testing to be set up and implemented. Consistent with this intuition, the synergies while still considerable, are smaller with larger values of α. As α becomes larger, testing capacity builds up more quickly so, on the margin, the value of delaying infections by using NPIs is smaller.

We now discuss the synergy between NPIs and testing/quarantining on economic outcomes. Figure 3 displays the cumulative drop in consumption as a function of γ. The blue line correspond to the equilibrium with testing and quarantining. The dashed-black, pink and green lines correspond to α equal to 0.0025, 0.005 and 0.02, respectively.

The impact of testing and quarantining on the economic costs of the epidemic gets dramatically larger as γ gets smaller, i.e. as there is more intensive use of NPIs. Suppose that $\alpha = 0.0025$. When $\gamma = 3$, testing and quarantining reduces the cumulative drop in consumption from 570 percent to 530 percent. When $\gamma = 0.3$, this drop goes from 520 percent

to 340 percent. So, the impact of testing and quarantining is much larger when γ is smaller.

The synergies in consumption mirror the synergies in health outcomes as people adjust their economic decisions in response to different infection risks.

5 Virus mutations and temporary immunity

A key maintained assumption of the economics literature on epidemics is that people who have recovered from the disease can't be reinfected. There are two reasons why this assumption might be incorrect. First, according to the World Health Organization (2020), there is no hard evidence that people acquire permanent immunity after recovering from Covid. Indeed, there is evidence that people do not acquire permanent immunity after exposure to other corona viruses (see, e.g., Shamanand and Galanti (2020)). Second, there have been instances of people who recovered from Covid but were then reinfected by a mutation of the original virus (see e.g. Resende et al. (2021)).[14]

In this section, we accomplish two objectives. First, we extend our model to allow for the possibility that recovered people can be reinfected. Second, we examine the efficacy of testing and quarantining and NPIs under those circumstances.

5.1 People outside the testing pool

People outside the testing pool maximize their lifetime utility, (10), subject to the budget constraint, (11), the transmission function, (12), and the probability of being infected, (13). The equation for the probability of being susceptible, (14), is replaced by the following equation

$$p(s_{t+1}|a_{t+1})[1-\pi_d p(i_t|a_t)] = p(s_t|a_t)(1-\tau_t^u) + \pi_s p(r_t|a_t).$$

Here, π_s denotes the probability that a recovered agent becomes susceptible again. In the standard SIR model $\pi_s = 0$. We add the following equation for $p(r_{t+1}|a_{t+1})$[15]

$$p(r_{t+1}|a_{t+1})[1-\pi_d p(i_t|a_t)] = p(r_t|a_t)(1-\pi_s) + \pi_r p(i_t|a_t).$$

The term $(1-\pi_s)p(r_t|a_t)$, is the probability that a person who is recovered does not lose immunity and remains recovered at time $t+1$.

[14] See Atkeson (2021) for a discussion of the impact of the U.K. variant of the virus on the course of the Covid epidemic.

[15] In the version of the model without reinfections, we replaced $p(r_{t+1}|a_{t+1})$ by $1-p(s_{t+1}|a_{t+1})-p(i_{t+1}|a_{t+1})$ instead of imposing the equation for $p(r_{t+1}|a_{t+1})$ as a constraint.

The first-order conditions for the problem of a person outside the testing pool are displayed in the appendix.

5.2 People inside the testing pool

The problem of people inside the testing pool remains the same as before with one important exception. The lifetime utility of a recovered person now takes into account the probability of becoming susceptible,

$$U_t^r = u(c_t^r, n_t^r, m_t^r) + \beta(1-\pi_s)U_{t+1}^r + \beta\pi_s U_{t+1}^s. \tag{26}$$

A recovered person maximizes (26) subject to the budget constraint (15). The first-order conditions for consumption and hours worked for a recovered person are the same as in the problem without reinfections.

5.3 Population dynamics

The equations governing population dynamics are the same as in the model without reinfections with the following exceptions. Equations (22), (23), (24), and (25) are replaced by

$$\begin{aligned}
S_{t+1}^k &= S_t^k - T_t^k + \pi_s R_t^k + \alpha(S_t^u - T_t^u + \pi_s R_t^u), \\
S_{t+1}^u &= (1-\alpha)(S_t^u - T_t^u + \pi_s R_t^u), \\
R_{t+1}^k &= R_t^k + \pi_r I_t^k - \pi_s R_t^k + \alpha\left(R_t^u + \pi_r I_t^u - \pi_s R_t^u\right), \\
R_{t+1}^u &= (1-\alpha)(R_t^u + \pi_r I_t^u - \pi_s R_t^u).
\end{aligned}$$

The economy converges asymptotically to a steady state in which the number of susceptible people and the ratio of infected people to recovered people are constant. These properties are straightforward to establish analytically for a simple SIR version of our model ($\pi_1 = \pi_2 = 0$) in which the following condition holds: $\pi_s < \pi_r + \pi_d$. To solve for the steady of the model with π_1 and π_2 different from zero, we guess and verify that there is a steady state in which S_t^k, S_t^u, I_t^k/R_t^k, and I_t^u/R_t^u are constant.

5.4 Quantitative results

As far as we know, there are no reliable estimates of the rate at which recovered people get reinfected by the virus that causes Covid. For this reason, we rely on estimates of reinfection

rates for the severe acute respiratory syndrome (SARS) to calibrate our model. Wu et al. (2007) report that SARS antibodies last on average for two years. Since each period in our model corresponds to a week, we choose $\pi_s = 1/(2 \times 52)$.

Figure 7 displays our results for $\gamma = 1$. The blue line, reproduced from Figure 2, and the red-dashed line corresponds to the model without and with reinfections, respectively. The key result is that, when π_s is positive, the number of infected people does not converge to zero. Since some recovered people become susceptible again, the asymptotic ratio of susceptible people to the initial population is 61 percent instead of 47 percentage in the no-reinfection economy. Critically, over a ten-year period the cumulative death toll is more than four times higher than in the no-reinfection economy.

The dashed grey line in Figure 7 displays the dynamics of the epidemic with reinfections and testing/quarantining ($\alpha = 0.005$). The latter policy substantially reduces the peak level of infections and the death toll from the epidemic. After 500 weeks, this death toll as a percentage of the initial population is 0.15 percent and 0.75 percent, with and without testing/quarantining, respectively. This difference is much larger than the analogue difference in the economy without reinfections. Without reinfections, the death toll from the epidemic as a percentage of the initial population is 0.1 percent and 0.2 percent, with and without testing/quarantining, respectively. Clearly, the benefits of testing and quarantining in terms of lives saved are even larger with than without reinfections.

Figure 7 also shows that testing and quarantining greatly reduces the severity of the recession. The peak-to-trough decline in consumption is roughly the same with or without testing and quarantining. The big difference is that, with reinfections, the economy never recovers unless there is testing and quarantining. Finally, it turns out that with reinfections, the synergies between PIs and testing/quarantining are relatively small. The reason is that infections keep recurring.

6 Conclusion

In this paper, we develop a SIR-based macroeconomic model in which people do not know their true health state. In this environment, testing allows the government to identify infected people and quarantine them. We argue that the potential social gains from such a policy are very large, especially when people adopt NPIs like mask use and social distancing. These NPIs slow down the spread of the virus, buying time for testing and quarantining to come

on line.

Policies like lockdowns and other restrictions to economic activity save lives but generate large recessions. In contrast, testing and quarantining, especially if combined with intensive use of NPIs, reduces both the death toll and the severity of the recession induced by the epidemic. The positive impact of testing and quarantining is particularly dramatic when people who recover from an infection acquire only temporary immunity to the virus.

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Appendix A Equilibrium Equations

This appendix provides the equilibrium equations for the model with unknown and known health states due to testing. We consider the model with temporary immunity. The model with permanent immunity is a special case where $\pi_s = 0$.

A.1 Equilibrium equations for people with unknown health states

Present value utility of people with unknown health states:

$$\begin{aligned} U_t^u &= u(c_t^u, n_t^u, m_t^u) + (1-\alpha)\beta\left[1-\pi_d p(i_t|a_t)\right] U_{t+1}^u \\ &\quad +\alpha\beta\left[1-\pi_d p(i_t|a_t)\right]\left[p(s_{t+1}|a_{t+1})U_{t+1}^s + p(i_{t+1}|a_{t+1})U_{t+1}^i + p(r_{t+1}|a_{t+1})U_{t+1}^r\right]. \end{aligned}$$

Transmission function, budget and probability transition functions:

$$\begin{aligned} \tau_t^u &= (1-m_t^u)\left\{\pi_1 c_t^u[I_t^u C_t^u\left(1-M_t^u\right) + \chi_t I_t^k C_t^i\left(1-M_t^i\right)] + \pi_2 n_t^u[I_t^u N_t^u\left(1-M_t^u\right) + \chi_t I_t^k N_t^i\left(1-M_t^i\right)]\right. \\ &\quad \left.+\pi_3\left(I_t^u\left(1-M_t^u\right) + I_t^k\left(1-M_t^i\right)\right)\right\}, \end{aligned}$$

$$c_t^u = An_t^u + \Gamma_t,$$

$$p(i_{t+1}|a_{t+1})[1-\pi_d p(i_t|a_t)] = \tau_t^u p(s_t|a_t) + (1-\pi_r-\pi_d)p(i_t|a_t),$$

$$p(s_{t+1}|a_{t+1})[1-\pi_d p(i_t|a_t)] = p(s_t|a_t)(1-\tau_t^u) + \pi_s p(r_t|a_t),$$

$$p(r_{t+1}|a_{t+1})[1-\pi_d p(i_t|a_t)] = p(r_t|a_t)(1-\pi_s) + \pi_r p(i_t|a_t).$$

First-order condition for c_t^u:

$$(1-\epsilon_t^c)\frac{1}{c_t^u} - \lambda_{bt}^u + \lambda_{\tau t}^u\left(1-m_t^u\right)\pi_1\left(I_t^u C_t^u\left(1-M_t^u\right) + \chi_t I_t^k C_t^i\left(1-M_t^i\right)\right) = 0.$$

First-order condition for n_t^u :

$$-\theta n_t^u + \lambda_{bt}^u A + \lambda_{\tau t}^u\left(1-m_t^u\right)\pi_2\left(I_t^u N_t^u\left(1-M_t^u\right) + \chi_t I_t^k N_t^i\left(1-M_t^i\right)\right) = 0.$$

First-order condition for m_t^u:

$$-\gamma m_t^u = \lambda_{\tau t}^u \frac{\tau_t^u}{1-m_t^u}$$

First-order condition for τ_t^u:

$$-\lambda_{\tau t}^u + \lambda_{it}^u p(s_t|a_t) - \lambda_{st}^u p(s_t|a_t) = 0.$$

First-order condition for $p(i_{t+1}|a_{t+1})$

$$\begin{aligned}&\frac{dU_t^u}{dp(i_{t+1}|a_{t+1})}\frac{1}{1-\pi_d p(i_t|a_t)}-\lambda_{it}^u+\lambda_{it+1}^u\beta p(i_{t+2}|a_{t+2})\pi_d\\&+\lambda_{it+1}^u\beta(1-\pi_r-\pi_d)+\lambda_{st+1}^u\beta\pi_d p(s_{t+2}|a_{t+2})\\&+\lambda_{rt+1}^u\beta\pi_d p(r_{t+2}|a_{t+2})+\lambda_{rt+1}^u\beta\pi_r.\end{aligned}$$

First-order condition for $p(s_{t+1}|a_{t+1})$

$$\frac{dU_t^u/dp(s_{t+1}|a_{t+1})}{1-\pi_d p(i_t|a_t)}+\lambda_{it+1}^u\beta\tau_{t+1}^u-\lambda_{st}^u+\lambda_{st+1}^u\beta(1-\tau_{t+1}^u)=0.$$

First-order condition $p(r_{t+1}|a_{t+1})$

$$\frac{dU_t^u}{dp(r_{t+1}|a_{t+1})}\frac{1}{1-\pi_d p(i_t|a_t)}+\lambda_{st+1}^u\beta\pi_s-\lambda_{rt}^u+\lambda_{rt+1}^u\beta(1-\pi_s)=0.$$

The relevant derivatives of lifetime utility are given by

$$\begin{aligned}&\frac{dU_t^u}{dp(i_{t+1}|a_{t+1})}=\alpha\beta\left[1-\pi_d p(i_t|a_t)\right]U_{t+1}^i-\left[(1-\alpha)\beta\right]^2\left[1-\pi_d p(i_t|a_t)\right]\pi_d U_{t+2}^u\\&-\pi_d\alpha(1-\alpha)\beta^2\left[1-\pi_d p(i_t|a_t)\right]\times\left[p(s_{t+2}|a_{t+2})U_{t+2}^s+p(i_{t+2}|a_{t+2})U_{t+2}^i+p(r_{t+2}|a_{t+2})U_{t+2}^r\right],\end{aligned}$$

$$\frac{dU_t^u}{dp(s_{t+1}|a_{t+1})}=\alpha\beta\left[1-\pi_d p(i_t|a_t)\right]U_{t+1}^s,$$

$$\frac{dU_t^u}{dp(r_{t+1}|a_{t+1})}=\alpha\beta\left[1-\pi_d p(i_t|a_t)\right]U_{t+1}^r.$$

A.2 Equilibrium equations for people with known health states after testing

$$c_t^s=An_t^s+\Gamma_t,$$

$$c_t^i=An_t^i+\Gamma_t^i,$$

$$c_t^r=An_t^r+\Gamma_t,$$

$$U_t^s=u(c_t^s,n_t^s,m_t^s)+\beta\left[(1-\tau_t^s)U_{t+1}^s+\tau_t^s U_{t+1}^i\right],$$

$$\begin{aligned}\tau_t^s\;=\;&(1-m_t^s)\left\{\pi_1 c_t^s[I_t^u C_t^u(1-M_t^u)+\chi_t I_t^k C_t^i\left(1-M_t^i\right)]+\pi_2 n_t^s[I_t^u N_t^u(1-M_t^u)+\chi_t I_t^k N_t^i\left(1-M_t^i\right)]\right.\\&\left.+\pi_3[I_t^u(1-M_t^u)+I_t^k\left(1-M_t^i\right)]\right\},\end{aligned}$$

$$(1-\epsilon_t^c)\frac{1}{c_t^s}-\lambda_{bt}^s+\lambda_{\tau t}^s(1-m_t^s)\pi_1\left(I_t^u C_t^u(1-M_t^u)+\chi_t I_t^k C_t^i\left(1-M_t^i\right)\right)=0,$$

$$-\theta n_t^s+A\lambda_{bt}^s+\lambda_{\tau t}^s(1-m_t^s)\pi_2\left(I_t^u N_t^u(1-M_t^u)+\chi_t I_t^k N_t^i\left(1-M_t^i\right)\right)=0,$$

$$-\gamma m_t^s = \lambda_{\tau t}^s \frac{\tau_t^s}{1 - m_t^s},$$

$$\beta \left(U_{t+1}^i - U_{t+1}^s \right) - \lambda_{\tau t}^s = 0,$$

$$U_t^i = u(c_t^i, n_t^i, m_t^i) + \beta \left[(1 - \pi_r - \pi_d) U_{t+1}^i + \pi_r U_{t+1}^r \right],$$

Without quarantining, the first-order conditions for c_t^i, n_t^i, and m_t^i are

$$u_1(c_t^i, n_t^i, m_t^i) = \lambda_{bt}^i,$$

$$u_2(c_t^i, n_t^i, m_t^i) = -A\lambda_{bt}^i,$$

$$-\gamma m_t^i \leq 0.$$

With quarantining

$$c_t^i = c_t^r,\ n_t^i = 0,\ m_t^i = m_t^s.$$

$$U_t^r = u(c_t^r, n_t^r, m_t^r) + \beta(1 - \pi_s) U_{t+1}^r + \beta \pi_s U_{t+1}^s,$$

$$(1 - \epsilon_t^c) \frac{1}{c_t^r} = \lambda_{bt}^r,$$

$$-\theta n_t^r = -A\lambda_{bt}^r,$$

Without quarantines the first-order condition for m_t^r is

$$-\gamma m_t^r \leq 0.$$

With quarantines

$$m_t^r = m_t^s.$$

A.3 Population dynamics

The present below the equations that govern population dynamics. The variable Pop_t^u denotes the total number of people outside of the testing pool at time t.

$$S_{t+1}^u = p(s_{t+1}|a_{t+1}) Pop_{t+1}^u,$$

$$I_{t+1}^u = p(i_{t+1}|a_{t+1}) Pop_{t+1}^u,$$

$$R^u_{t+1} = [1 - p(s_{t+1}|a_{t+1}) - p(i_{t+1}|a_{t+1})]Pop^u_{t+1},$$

$$D^u_{t+1} = D^u_t + \pi_d I^u_t,$$

$$T^u_t = \tau^u_t p(s_t|a_t) Pop^u_t,$$

$$Pop^u_{t+1} = Pop^u_t [1 - \pi_d p(i_t|a_t)] (1 - \alpha),$$

$$\begin{aligned} T^k_t &= (1 - M^s_t)\pi_1 S^k_t C^s_t [I^u_t C^u_t (1 - M^u_t) + \chi_t I^k_t C^i_t (1 - M^i_t)] \\ &\quad + (1 - M^s_t)\pi_2 S^k_t N^s_t [I^u_t N^u_t (1 - M^u_t) + \chi_t I^k_t N^i_t (1 - M^i_t)] \\ &\quad + (1 - M^s_t)\pi_3 S^k_t [I^u_t (1 - M^u_t) + I^k_t (1 - M^i_t)], \end{aligned}$$

$$S^k_{t+1} = S^k_t - T^k_t + \pi_s R^k_t + \alpha(S^u_t - T^u_t + \pi_s R^u_t),$$

$$I^k_{t+1} = T^k_t + (1 - \pi_r - \pi_d) I^k_t + \alpha [T^u_t + (1 - \pi_r - \pi_d) I^u_t],$$

$$R^k_{t+1} = R^k_t + \pi_r I^k_t - \pi_s R^k_t + \alpha (R^u_t + \pi_r I^u_t - \pi_s R^u_t),$$

$$D^k_{t+1} = D^k_t + \pi_d I^k_t.$$

A.4 Government budget and equilibrium

$$\left(S^k_t + R^k_t + S^u_t + I^u_t + R^u_t\right) \Gamma_t + I^k_t \Gamma^i_t = 0,$$

$$c^j_t = C^j_t, n^j_t = N^j_t.$$

A.5 Aggregate variables

$$\begin{aligned} C_t &= \left(S^k_t C^s_t + I^k_t C^i_t + R^k_t C^r_t\right) + \left(S^u_t + I^u_t + R^u_t\right) C^u_t, \\ N_t &= \left(S^k_t N^s_t + I^k_t N^i_t + R^k_t N^r_t\right) + \left(S^u_t + I^u_t + R^u_t\right) N^u_t, \end{aligned}$$

$$\begin{aligned} D_t &= D^u_t + D^k_t, \\ R_t &= R^u_t + R^k_t, \\ I_t &= I^u_t + I^k_t, \\ S_t &= S^u_t + S^k_t. \end{aligned}$$

A.6 Numerical algorithm

We use a time-stacking algorithm together with a gradient-based method to solve for the equilibrium paths of all endogenous variables for $t = 0, ..., 500$.

Figure 1: Data versus Model

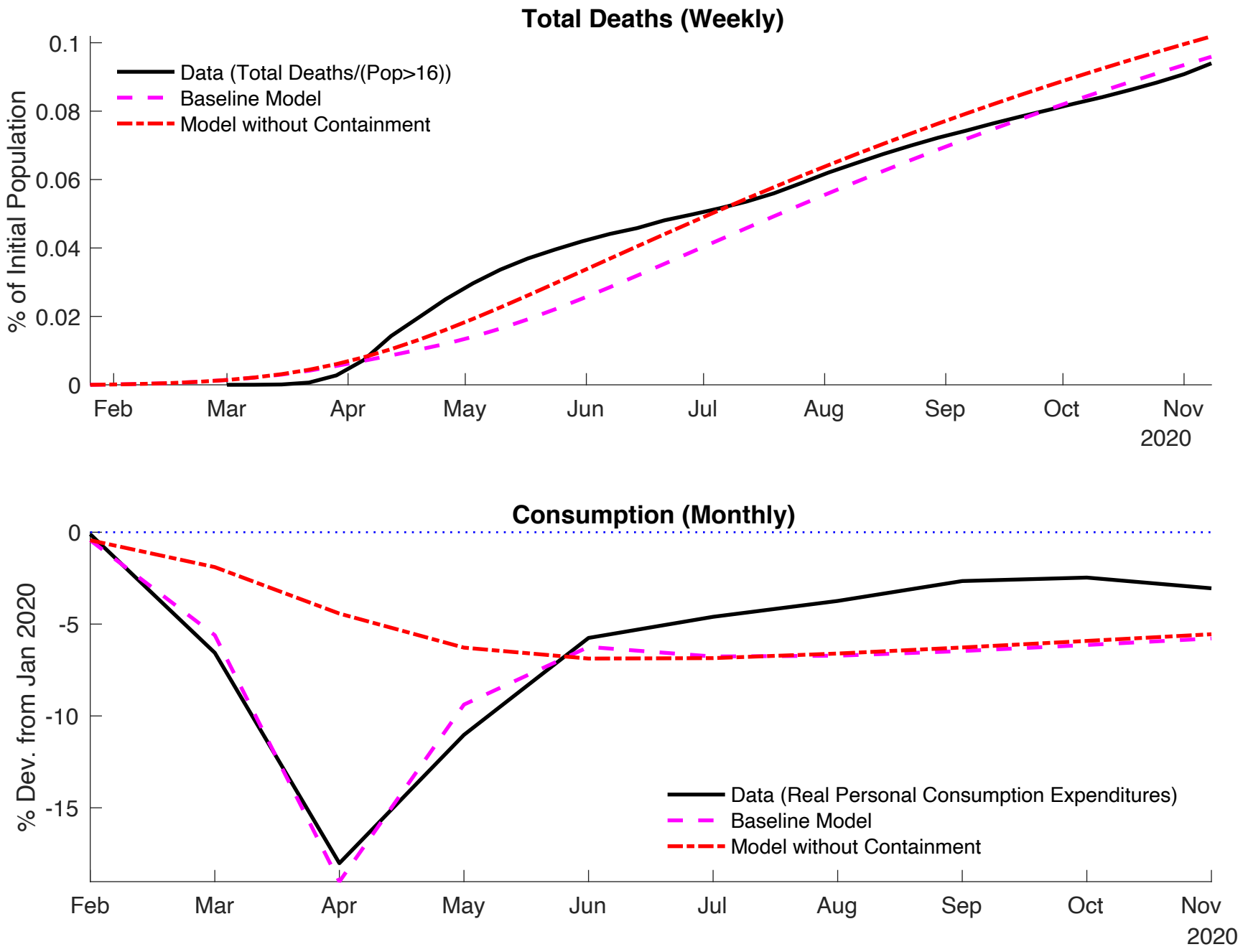


Figure 2: Effects of NPI Disutility in Model with Unknown Health Status

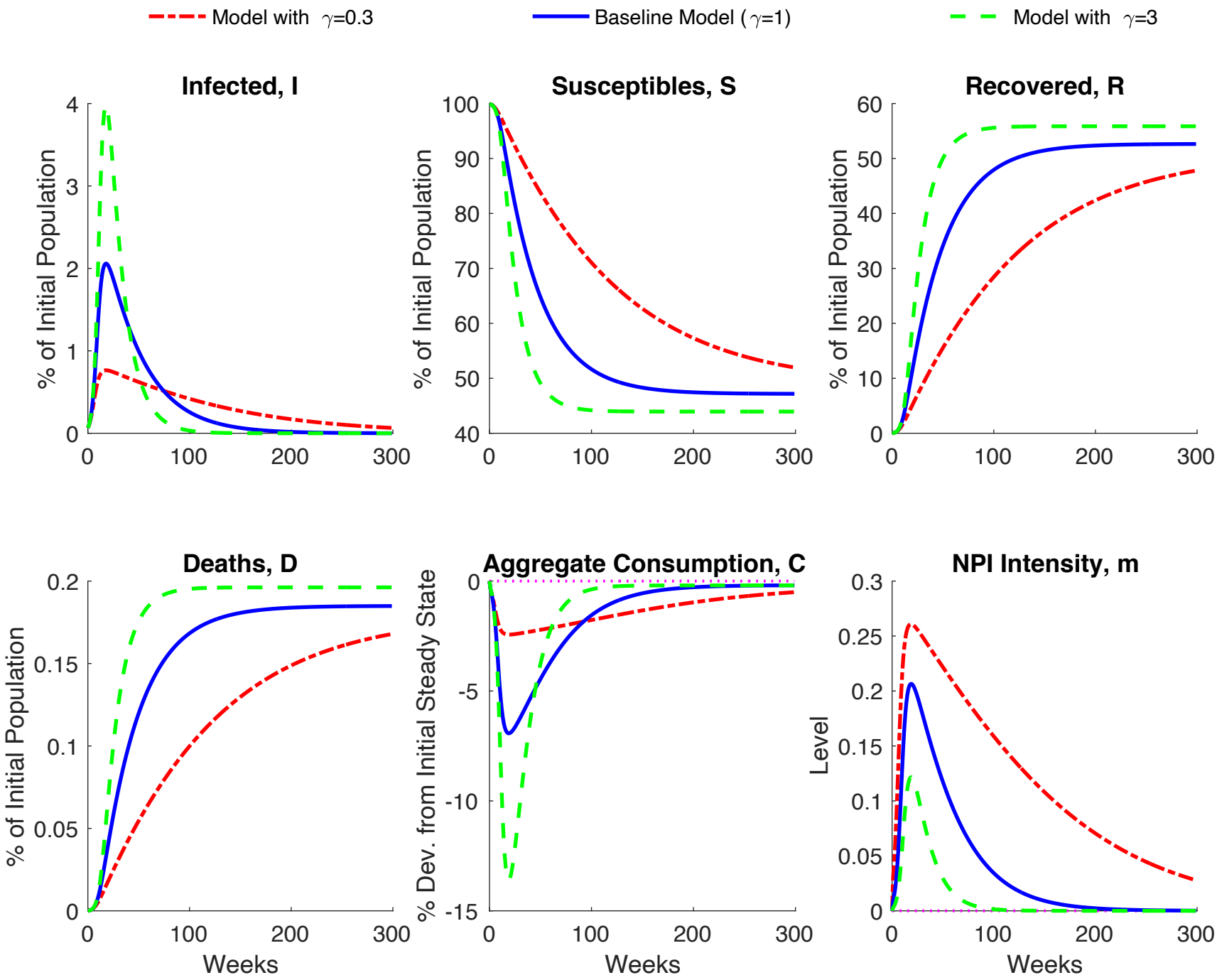


Figure 3: Effects of NPI, Testing and Quarantining

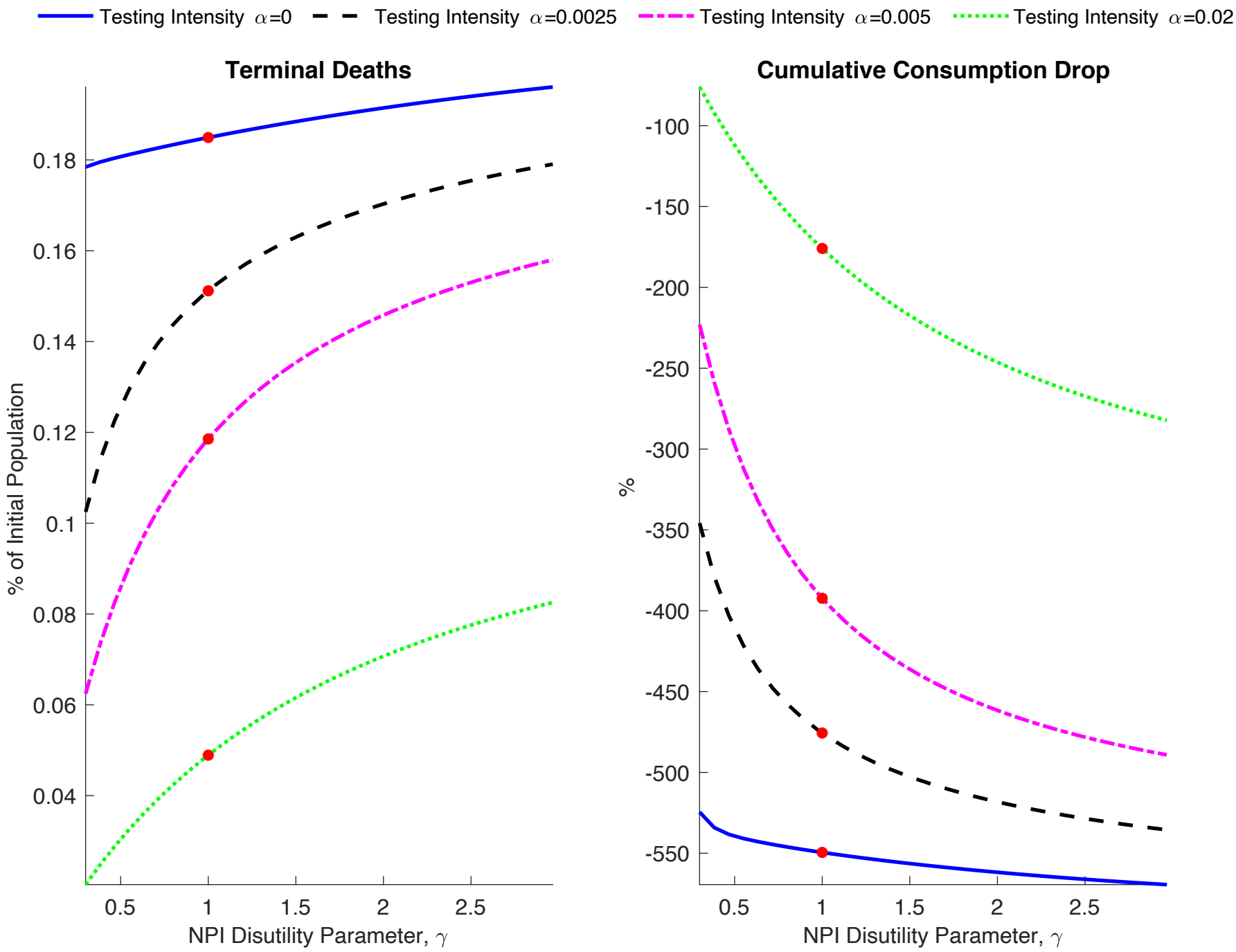


Figure 4: Model with Testing and Quarantines

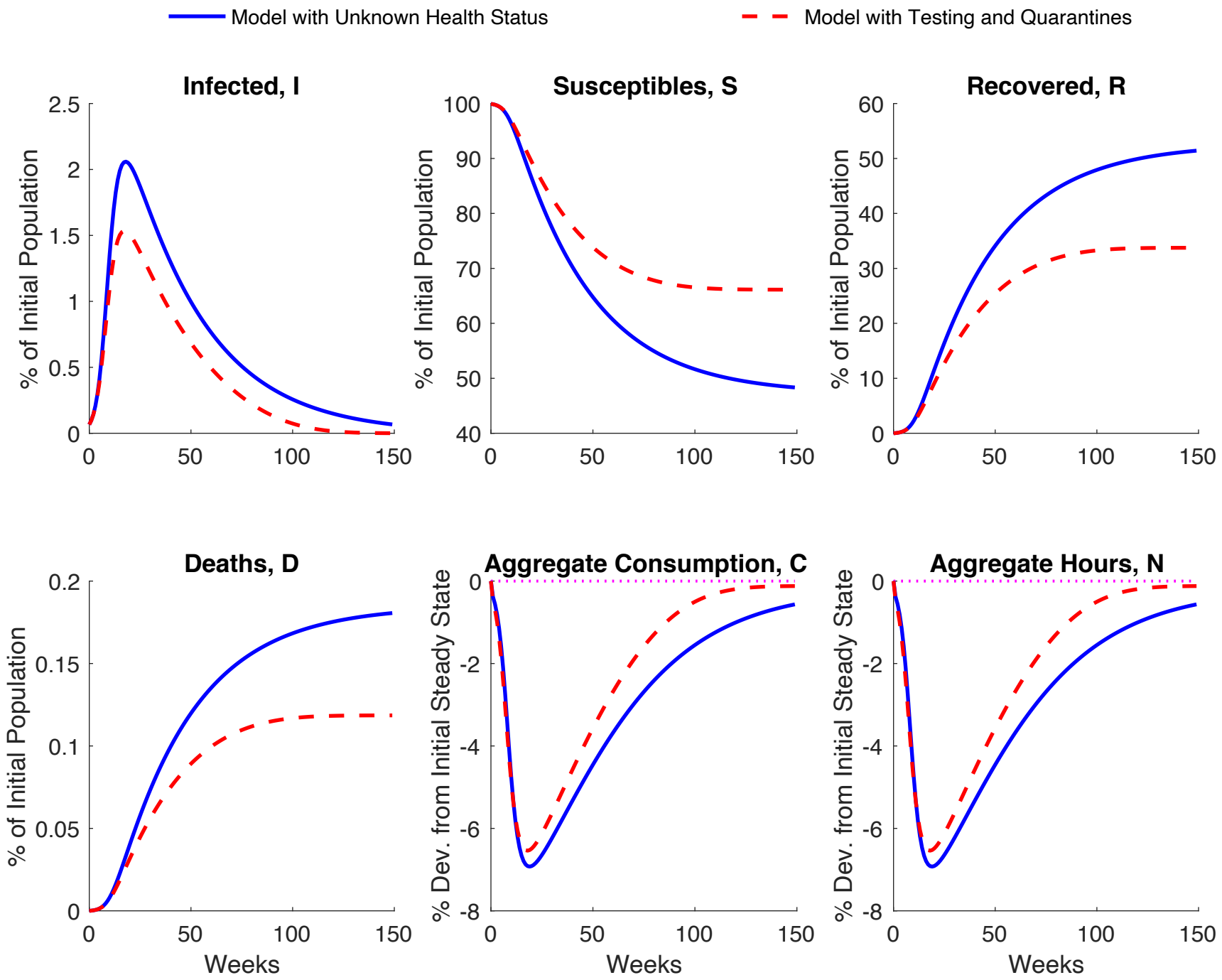


Figure 5: Model with Testing and Quarantines

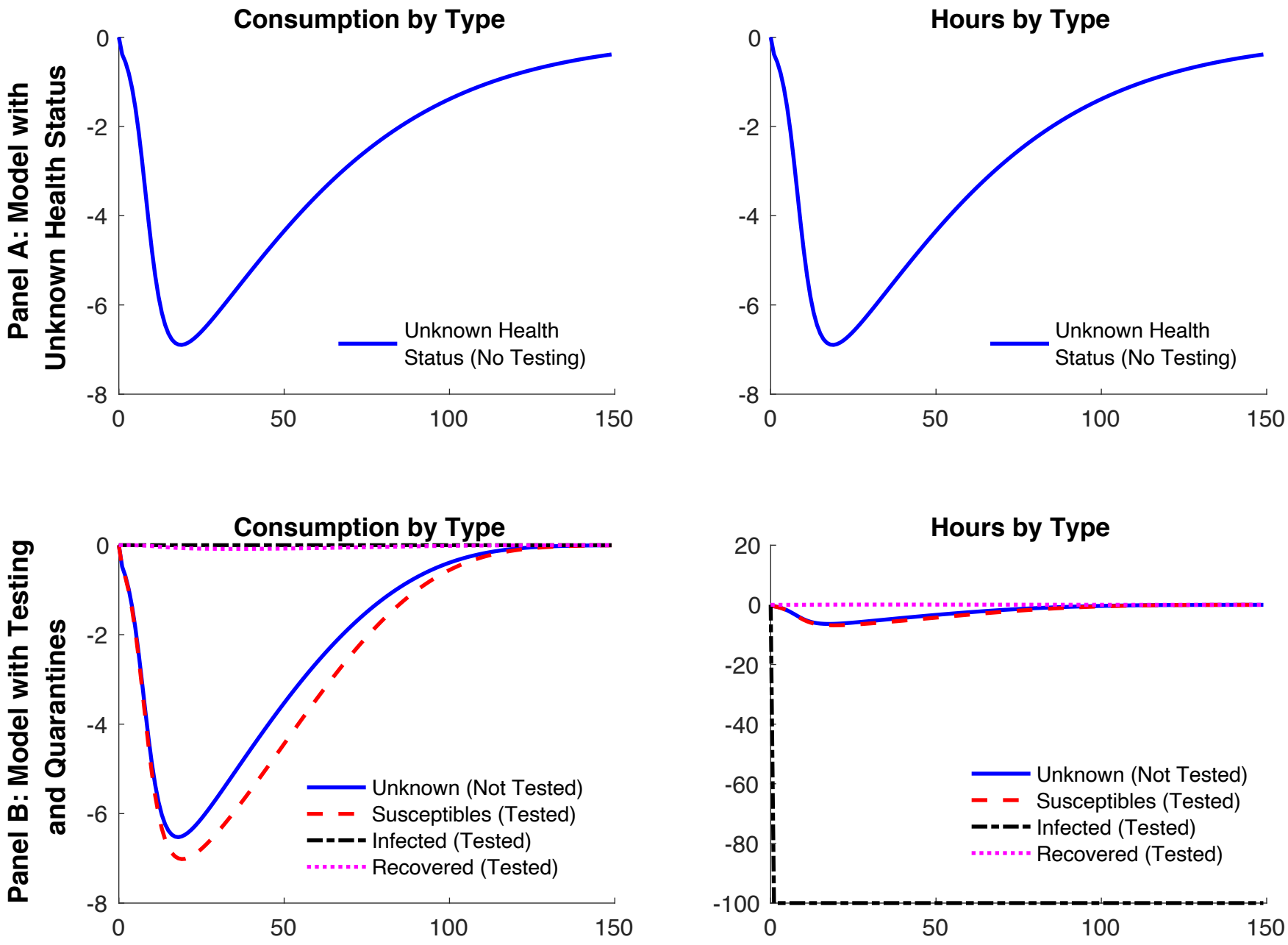
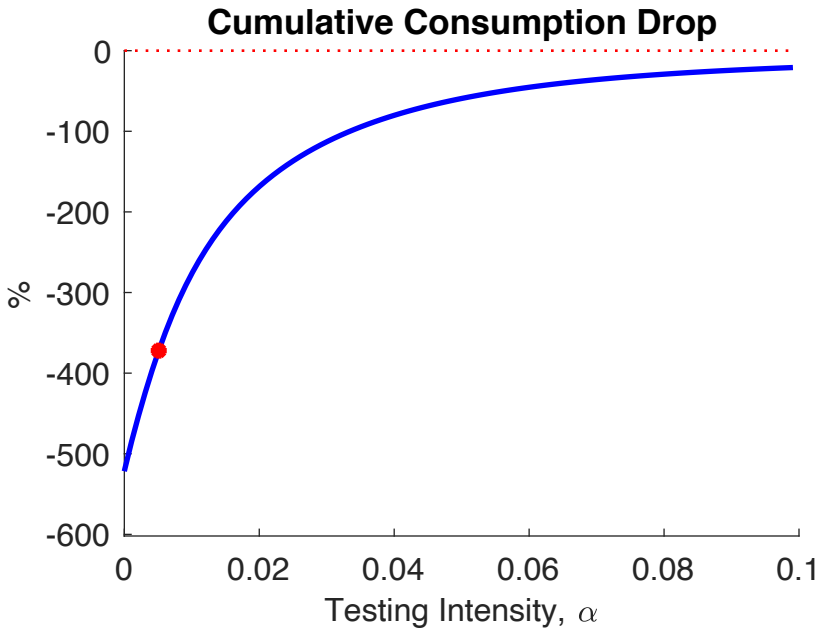
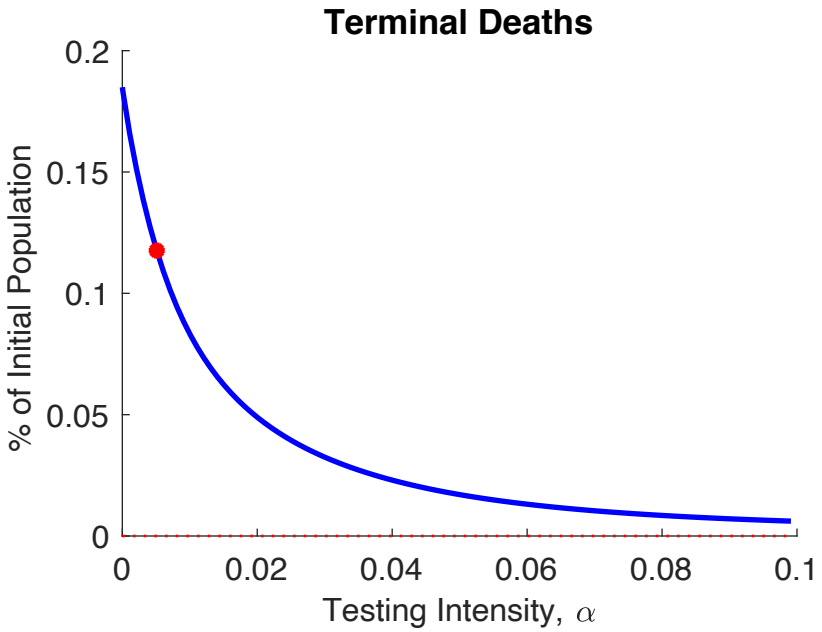


Figure 6: Model with Testing and Quarantines





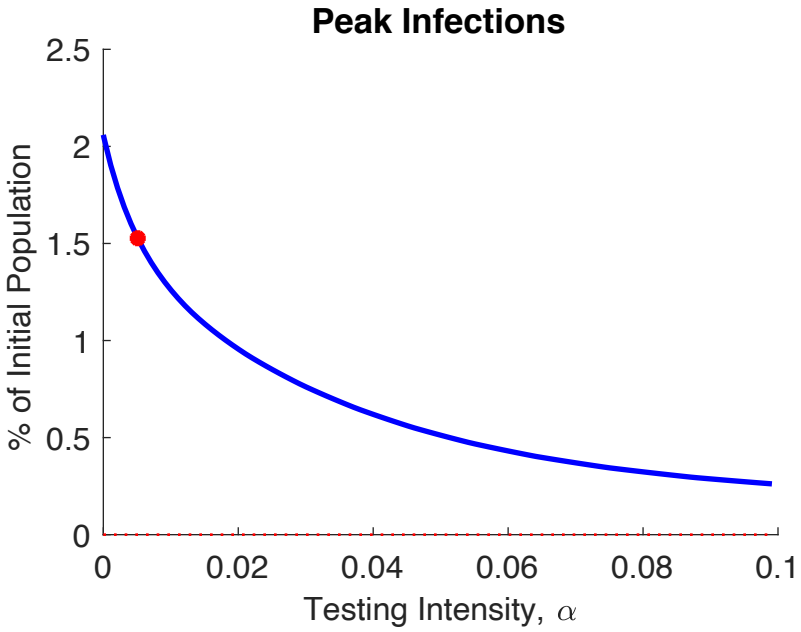


Figure 7: Model with Re-infections, Testing and Quarantines