Unemployment and Business Cycles

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Disclaimer: The views expressed are those of the authors and not necessarily those of the Federal Reserve Board or any other person associated with the Federal Reserve System.
Background

• Key challenge for modern business cycle models.
  – How to account for observed volatility of labor market variables?

• Standard view
  – For plausibly parameterized models, in a boom, wages rise too rapidly, limiting expansion of employment.
  – Classic RBC models, standard efficiency wage models (Alexopolous), standard DMP models (Shimer).
Ongoing Efforts

- Empirical NK models more successful in accounting for cyclical properties of employment
  - Assume wage setting is subject to nominal rigidities.

- Problems
  - Assume result: wages are exogenously sticky,
  - Agents wouldn’t choose to subject themselves to nominal wage frictions imposed on them by the modeler.
  - Can’t use models to examine some key policy issues, e.g. extension of unemployment benefits.
  - Wages are always changing because of indexation.
What We Do

• Develop and estimate variants of search and matching model that accounts for key macro aggregates, including labor market variables.

• We derive wage inertia as an *equilibrium* outcome.

• *AOB model*: modified version of Hall-Milgrom (2008, HM):
  – When workers and firms bargain, they think they’re better off reaching agreement than parting ways.
  – Disagreement leads to continued negotiations.
  – If negotiation costs don’t depend sensitively on state of economy, neither do wages.

• Nash *model*: real wages determined by a Nash sharing rule.
What We Do

- Embed AOB and Nash setups in empirical, dynamic GE model.

- Price setting is subject to exogenous nominal Calvo-style rigidities.
  - No price indexation.

- Estimate models using Bayesian variant of strategy in CEE (2005).

- Minimize distance between model and exactly identified VAR-based dynamic response to monetary policy and technology shocks.
  - 12 post-war, quarterly U.S. times series that include key labor market variables.
Wage Inertia

• Real wages have two key properties which together define wage inertia.
  – Relatively small contemporaneous response of real wage to shocks.
  – Response that does occur is very persistent.

• Wage inertia is critical element of model’s ability to account for estimated response of the economy to shocks, particularly inflation.
Key Model Property

- Monetary policy shock:
  - In data, small response of inflation.
  - In model, small response of wages leads to small response of marginal cost and inflation.

- Neutral technology shock:
  - In data: relatively large drop in inflation.
  - In model, other things equal, a positive, neutral technology shock leads to a relatively large drop in inflation.
  - If wages rose by too much it would undo the effect of the technology shock on marginal costs.
  - So if real wage rose by a lot, model couldn’t account for observed drop in inflation.
Model Comparisons

- Estimated AOB model outperforms the estimated Nash model in terms of marginal likelihood of the data.

- At posterior mode of the parameters, AOB and Nash models generate impulse response functions that are virtually identical to each other.

- But to do this, Nash model needs very high replacement ratio (0.90) that’s extremely implausible from the perspective of our prior.

- AOB model doesn’t require an implausible value for any of model parameters.

- That’s why marginal likelihood of AOB model is so much higher than that of Nash model.
Wage inertia

• Wage inertia is central to the success of our search and matching models.

• Is it a central property of a broader class of empirically successful models?

• In our models, real wage is solution to a bargaining problem, whose solution is a surplus sharing rule.

• That rule can be interpreted as restricted rule for setting real wage rate as function of model’s date $t$ state variables.
Wage inertia...

- Estimate model where we replace AOB sharing rule with unrestricted real wage rules.

- General real wage rule: real wage is unrestricted function of all of model’s date $t$ state variables.

- Estimated rule exhibits wage inertia.
  - Real wage responds relatively little to shocks.
  - Response that does occur is very persistent.

- We also report a parsimonious version of generalized real wage that does about as well empirically.
Comparisons: Empirical NK Sticky Wage Model

- Model outperforms sticky wage models: EHL, no indexation.

- Statistical fit of sticky wage model with indexation is about same as AOB model.

- Given the limitations of sticky wage models, there’s simply no need to work with them.

- AOB model fits the data better, and can be used to analyze a broader set of labor market variables and set of policy questions.
Changes in Unemployment Benefits

• Critical interaction between nominal rigidities, monetary policy and effects of a change in unemployment benefits.
  – Normal times: monetary policy *amplifies* contractionary effects of increase in unemployment benefits.
  – When ZLB binds, monetary policy *mitigates* contractionary effects associated with increase in unemployment benefits.
    • Increase in unemployment benefits can actually be expansionary.

• Actual effects of increase in unemployment benefits in ZLB are likely to be quite small.
Labor Market Model

- Large number of identical and competitive firms; produce homogeneous output using only labor, $l_t$.

- Firm that wishes to meet a worker in period $t$ must post a vacancy at cost $s_t$, expressed in units of the consumption good.

- The vacancy is filled with probability $Q_t$.

$$Q_t = \sigma_m \Gamma_t^{-\sigma}$$

$$\Gamma_t = \frac{\nu_t l_{t-1}}{1 - \rho l_{t-1}}$$

- If a vacancy is filled, firm must pay a fixed real cost, $\kappa_t$, before bargaining with newly-found worker.
Worker and firm engage in alternating offer bargaining.

Upon agreement, production begins immediately.

Job continues in next period with probability, $\rho$. 
Value Functions

- $J_t$ is the value to a firm of an employed worker:

\[ J_t = \vartheta_t - w_t + \rho E_t m_{t+1} J_{t+1}. \]

- $\vartheta_t$ and $m_{t+1}$ are determined in general equilibrium.

- Free entry and zero profits dictate:

\[ Q_t (J_t - \kappa_t) = s_t. \]
Value Functions

- Value of employment to a worker:

\[ V_t = w_t + E_t m_{t+1} \left[ \rho V_{t+1} + (1 - \rho) \left( f_{t+1} V_{t+1} + (1 - f_{t+1}) U_{t+1} \right) \right]. \]

where \( f_{t+1} V_{t+1} \) are job-to-job transitions.

- Employment law of motion and job finding rate:

\[ l_t = (\rho + x_t) l_{t-1} \text{ and } f_t = \frac{x_t l_{t-1}}{1 - \rho l_{t-1}} \]

where \( x_t \) denotes the hiring rate.
Value Functions

- Value of unemployment to a worker:

\[ U_t = D + E_t m_{t+1} \left[ f_{t+1} V_{t+1} + (1 - f_{t+1}) U_{t+1} \right]. \]

where \( D \) denotes unemployment benefits.
Alternating Offer Bargaining

- Bargaining occurs via a sequence of offers and counter offers.

- Baseline specification:
  - Bargain over current wage rate between firms and workers.
  - Agents take outcome of future wage bargains as given.

- Bargaining occurs between two types of workers and firms
  - Those that just met for the first time.
  - Those that reached an agreement in previous period and match survived.
Alternating Offers

• Each quarter is divided into $M$ equal subperiods, $m = 1, .., M$.
  – Firm makes an opening wage offer in $m = 1$.
  – Worker can reject and make a counter offer in $m = 2$.
  – Firm can reject worker’s wage offer and make a new offer in next sub-period,...
  – If we get there because of rejections, worker makes a take-it-or-leave-it offer in last subperiod $M$.

• If an offer is accepted in any sub-period $m$, production begins immediately.
  – Value of production in any subperiod is $\vartheta_t / M$.

• Solution to the problem:

$$w_t^1 (\equiv w_t), w_t^2, ..., w_t^M.$$
Alternating Offers: round 1

• Firm opens bargaining with an offer, $w_t^1$.

• Worker can choose one of three responses to a firm’s offer:
  1. Accept offer, $w_t^1$. Get utility $V_t^1$.
  3. Reject offer. Intend to propose counter offer.
     – With probability $\delta$ negotiations break down. Go to $U_t^1$.
     – Otherwise, make counter offer $w_t^2$. Utility of this option is $V_t^2$.

• In equilibrium (3) is preferred to (2).
Firm’s Offer: round 1

- Firm offers $w^1_t$ as low as possible subject to worker not rejecting it.

- Optimization by firm leads to:

\[
\text{utility of worker who accepts firm offer and goes to work} \quad \hat{V}^1_t = \delta U_t^1 + (1 - \delta) \left( \frac{D}{M} + V^2_t \right) \\
\text{utility of worker who rejects firm offer and intends to make counteroffer}
\]
Alternating Offers: round 2

- Suppose the worker rejected wage offer \( w^1_t \) and makes a counter offer, \( w^2_t \).
- Firm can choose one of three responses to a workers’ counter offer:
  1. Accept offer, \( w^2_t \).
  - Firm value \( J^2_t \) reflects that time \( t \) marginal revenue product is \( \vartheta_t \frac{M-1}{M} \).
  3. Reject offer. Intend to propose counter offer.
     - With probability \( \delta \) negotiations break down. Go to zero value.
     - Otherwise, make counter offer \( w^3_t \). Utility of this option is \( J^3_t \).

- In equilibrium (3) is preferred to (2).
Worker Offer: round 2

- Worker proposes highest possible wage $w_t^2$ subject to firm not rejecting it.

\[
J_t^2 = \begin{cases} \text{value of firm that accepts worker offer} \\ \text{value of firm that rejects worker offer and intends to make counteroffer} \end{cases} = \delta \times 0 + (1 - \delta) \left[-\gamma + J_t^3\right]
\]

- The firm incurs cost $\gamma$ to make a counter offer.

- Firm value $J_t^2$ reflects that time $t$ marginal revenue product is $\theta_t \frac{M-2}{M}$.
Alternating Offers, Final Round

- Each bargaining round requires the wage for the next round.
- In the last round, the worker makes a final, take-it-or-leave-it-offer:

\[
\text{value of firm that accepts worker offer in last round} = \text{value of firm that rejects worker's take-it-or-leave-it offer}
\]

\[
J_t^M = 0
\]

- Firm value \( J_t^M \) reflects that time \( t \) marginal revenue product is \( \vartheta_t \frac{1}{M} \).
Alternating Offers

• Doesn’t correspond to ‘in-the-hut’ bargaining.

• Outside option influences workers’ disagreement payoff, negotiated wage rate.

• Business cycle affects bargaining via $U_t, V_t$ and $J_t$.
  – Monetary policy shock, technology shocks,....

• Responsiveness depends on parameters of alternating offers, replacement rate, ...
Calculations

- $M$ equilibrium conditions for the $M$ unknowns
  - $w_t \equiv w_t^1, w_t^M, w_t^{M-1}, w_t^{M-2}, \ldots, w_t^2$.

- Computations reduce to:
  - one equation determining expected present value of wages,
  - one equation determining spot wage.

- Potential non-uniqueness in spot wage rate resolved by assuming that wages in each date are time-invariant function of minimal set of state variables.
Comparisons

- Sharing rule implied by AOB model: rule for setting real wage rate as a function of model’s date $t$ state variables.

- AOB model:

$$J_t = \beta_1 (V_t - U_t) - \beta_2 \gamma + \beta_3 (\vartheta_t - D),$$

- Nash model:

$$J_t = \frac{1 - \eta}{\eta} (V_t - U_t)$$

- Note there are two constant terms in AOB model involving $\gamma$ and $D$ that aren’t a function of state of the economy.
Alternative Bargaining Arrangements

- Allocation we’re studying is identical to one in which agents bargain only once over expected present value of wages.

- Rationalizes worker having fixed nominal wage as long as he’s with firm.

- Wages of new hires more volatile than wages of incumbents.

- Key issue associated with this bargaining structure: commitment and time consistency.
Final Goods Producers

- Competitive final goods production

\[ Y_t = \left[ \int_{1}^{\frac{1}{\lambda_f}} \frac{1}{\lambda_f} dJ \right]^{\lambda_f} \]

- \( Y_t \) can be used to produce either consumption goods or investment goods.

- Production of investment good uses a linear technology in which one unit of \( Y_t \) is transformed into \( \Psi_t \) units of \( I_t \).
Retailers

- $j^{th}$ input produced by monopolistic ‘retailers’:
  \[ Y_{j,t} = k_{j,t} \alpha \left( z_{t} h_{j,t} \right)^{1-\alpha} - \phi_{t}. \]

- $h_{j,t}$ is quantity of an intermediate good purchased by the $j^{th}$ retailer.
  - Purchased in competitive markets for real price, $\vartheta_{t}$.

- This good is purchased in competitive markets at price $P_{t}^{h}$ from a wholesaler.

- Retailer must borrow $P_{t}^{h} h_{j,t}$ at gross nominal interest rate, $R_{t}$.

- Retailer repays loan at end of period $t$ after receiving sales revenues.
Retailers and wholesalers

- Retailers choose prices subject to Calvo sticky price frictions (no price indexation).

\[ P_{j,t} = \begin{cases} 
  P_{j,t-1} & \text{with probability } \zeta \\
  \tilde{P}_t & \text{with probability } 1 - \zeta 
\end{cases} \]

- Don’t allow non-optimizing firms to index their prices to some measure of inflation.

- Wholesalers firms produce intermediate good using labor which has a fixed marginal productivity of unity.

- Hire workers in labor market model discussed above.
DSGE Model

- Representative household:

\[ E_0 \sum_{t=0}^{\infty} \beta^t \ln (C_t - bC_{t-1}), \ 0 \leq b < 1. \]

\[ P_tC_t + P_{I,t}I_t + B_{t+1} \leq \]

\[ (R_{K,t}u^K_t - a(u^K_t)P_{I,t})K_t + (1 - l_t)P_tD_t + W_tl_t + R_{t-1}B_t - T_t. \]

- Stock of capital evolves as follows

\[ K_{t+1} = (1 - \delta_K)K_t + [1 - S(I_t/I_{t-1})]I_t. \]
Monetary Policy

\[
\ln\left(\frac{R_t}{R}\right) = \rho_R \ln\left(\frac{R_{t-1}}{R}\right) + (1 - \rho_R) \left[ r_{\pi} \ln\left(\frac{\pi_t}{\bar{\pi}}\right) + r_y \ln\left(\frac{\mathcal{Y}_t}{\mathcal{Y}_t^*}\right)\right] + \sigma_R \varepsilon_{R,t}. \\

\mathcal{Y}_t = C_t + I_t / \Psi_t + G_t,
\]

• \(\mathcal{Y}_t^*\) denotes value of \(\mathcal{Y}_t\) along non-stochastic steady state growth path.
Estimated Medium-Sized DSGE Mode

- Estimate VAR impulse responses of aggregate variables to a monetary policy shock and two types of technology shocks.

- 11 variables considered:
  - Macro variables and real wage, hours worked, unemployment, job finding rate, vacancies.

- Estimate model using Bayesian variant of CEE (2005) strategy:
  - Minimizes distance between dynamic response to three shocks in model, analog objects in the data.
  - Particular Bayesian strategy developed in Christiano, Trabandt and Walentin (2011).
Identifying Assumptions at VAR stage

- Only variable that monetary policy shock affects *contemporaneously* is Federal Funds Rate.

- The only shocks that affect *labor productivity in long-run* are innovations to neutral technology and investment specific technology shocks.

- Only shock that affects *relative price of investment in long-run* is innovation to investment technology shock.
Posterior Mode of Key Parameters (AOB)

- Prices change on average every 4 quarters (no price indexation).

- $\delta$: roughly 0.20% chance of a breakup after rejection.

- $\gamma$: cost to firm of preparing counteroffer is $1/4$ of a day’s worth of production.

- It costs roughly 0.6 of one day’s revenue for a firm to prepare a counteroffer to a worker (depends on $\kappa$).
Fixed cost component of hiring accounts for the lion's share of the total cost of meeting a worker

- Fixed cost component of meeting a worker, expressed as a percent of total cost is:
  \[ \frac{\eta_h}{\eta_h + \eta_s} = 0.94. \]

- Total cost associated with hiring a new worker is roughly 7 percent of the wage rate
Posterior Mode of Key Parameters

- Replacement ratio is 0.37.

- HM report a range of estimates for the replacement ratio between 0.1 and 0.4.

- Gertler, Sala and Trigari (2008) : plausible range for replacement ratio is 0.4 to 0.7.
  - Upper boundary takes into account informal sources of insurance.

- Prior, posterior distributions of $D/w$ are quite similar.
  - This parameter doesn’t play critical role in AOB model’s ability to account for the data.
AOB: Monetary Shock

Figure 1: Responses to a Monetary Policy Shock: AOB vs. Calvo

Notes: x-axis: quarters, y-axis: percent
**Intuition**

- Expansionary monetary policy shock drives real interest rate down, inducing an increase in the demand for final goods.

- Induces increase in demand for output of sticky price retailers.

- Since they must satisfy demand, retailers purchase more of wholesale good.

- So relative price of wholesale good increases and MRP associated with a worker rises.

- Motivates wholesalers to hire more workers, increases probability that unemployed worker finds a job.

- Induces a rise in workers’ disagreement payoffs, generates rise in the real wage.
Intuition

- AOB generates moderate increase in real wages, a large rise in employment, a substantial decline in unemployment, and a small rise in inflation.

- If there was a large, persistent rise in the real wage, the model would generate a counterfactually large rise in inflation.

- Real wages are key component of firms’ real marginal costs.

- Firms that reset prices set those prices as increasing function of current and expected future real marginal cost.

- To account for observed cyclical behavior of inflation model it’s critical for model to exhibit wage inertia.
Figure 2: Responses to a Neutral Technology Shock: AOB vs. Calvo

Notes: x-axis: quarters, y-axis: percent
Nash Model

- Impulse response functions are virtually identical to the ones implied by the estimated AOB model.

- But posterior mode for $D/w$ is 0.88, posterior probability interval is very tight.

- Substantial 14 log point difference in marginal likelihood between the two models
  - Nash model has to reach far into right tail of prior distribution for $D/w$ to match the impulse response functions.

- High value of $D/w$ is critical to the performance of the Nash model.
  - Re-calculate Nash impulse response functions making only one change: set $D/w$, to 0.37.
  - Also re-estimate Nashm subject to constraint $D/W = 0.37$. 
Nash: Monetary Shock

Figure 4: Responses to a Monetary Policy Shock: Nash Bargaining

Notes: x-axis: quarters, y-axis: percent
Figure 5: Responses to a Neutral Technology Shock: Nash Bargaining

Notes: x-axis: quarters, y-axis: percent
Reduced form real wage models

- Wage inertia is central to the success of our search and matching models.

- Is it a central property of a broader class of empirically successful models?

- In our models, real wage emerges from bargaining problem, whose solution is a surplus sharing rule.

- That rule can be interpreted as restricted rules for setting real wage rate as function of model’s date $t$ state variables.

- Estimate versions of our model in which sharing rule is replaced by unrestricted real wage rules.
General Real Wage Rule

- Real wage is linear function of nine date $t$ state variables of the AOB model.
  \[
  \bar{w}_t \equiv \frac{w_t}{\Phi_t}
  \]

- Marginal likelihood is roughly 20 log points higher than it is for the estimated AOB model.

- Estimated general wage rule exhibits wage inertia.
  - Real wage responds relatively little to shocks.
  - Response that does occur is very persistent.
Reduced Form Real Wage Models
Simple Real Wage Rule

\[ \log \bar{w}_t = \text{constant} + \iota_1 \log \bar{w}_{t-1} + \iota_2 \log l_{t-1} + \iota_3 \log \mu_{z,t} + \iota_4 \log \mu_{\Psi,t}. \]

- Impact on \( \log w_t \) of innovation in \( \log z_t \) and in \( \log \Psi_t \) is \( 1 + \iota_3 \) and \( 1 + \iota_4 \alpha / (1 - \alpha) \).

- Negative values of \( \iota_3 \) and \( \iota_4 \) imply less than complete pass-through from technology shocks to real wage in period of shock.

- High values of \( \iota_1 \) ensure persistent incomplete pass-through.

- Anticipate a low value of \( \iota_2 \) because estimated response of \( w_t \) to monetary policy shock is persistently small.
Reduced form simple real wage rule

\[
\hat{w}_t = 0.96 \hat{w}_{t-1} + 0.03 \hat{l}_{t-1} - 0.28 \hat{\mu}_{z,t} - 0.26 \hat{\mu}_{\Psi,t}
\]

- Marginal likelihood is about 18 log points higher than it is for the estimated AOB model.
Reduced Form Real Wage Rules

Figure 7: Impulse Responses to Shocks: Simple and General Wage Rules

Notes: x-axis: quarters, y-axis: percent
Why bother with structural models?

- AOB model capture key features of reduced form real wage models.

- Can’t use reduced form models to study effects of policy interventions such as a change in unemployment benefits.
  - Coefficients in reduced form wage rules, including the constants, depend on objects like $D$.
  - Reduced form are silent on how these coefficients vary in response to changes in policy.
What about NK Sticky Wage Models?

Figure 1: Responses to a Monetary Policy Shock: AOB vs. Calvo

- GDP
- Unemployment Rate
- Inflation
- Federal Funds Rate
- Hours
- Real Wage
- Consumption
- Rel. Price Investment
- Investment
- Capacity Utilization
- Job Finding Rate
- Vacancies

Notes: x-axis: quarters, y-axis: percent
NK Sticky Wage Models

Figure 2: Responses to a Neutral Technology Shock: AOB vs. Calvo

<table>
<thead>
<tr>
<th>GDP</th>
<th>Unemployment Rate</th>
<th>Inflation</th>
<th>Federal Funds Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hours</td>
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<td>Vacancies</td>
</tr>
</tbody>
</table>

Notes: x-axis: quarters, y-axis: percent
Sticky Wage Model Comparisons

- AOB model and sticky nominal wage model don’t address the same data.
  - Sticky wage model has no implications for vacancies, job finding rate, unemployment.

- Integrate out unemployment, job finding rate, vacancies from marginal likelihood associated with AOB model.

- Integration is performed on Laplace approximation of posterior distribution.
  - We provide evidence on quality of approximation.

- Marginal likelihood for AOB model is about 60 log points higher than it is for sticky nominal wage model.
Sticky Wage Model with Indexation

• Also estimated sticky nominal wage model with indexation.
  – If labor supplier can’t re-optimize his wage, it changes by the steady state growth rate of output times the lagged inflation rate.

• Impulse response functions of AOB model and this sticky wage model are qualitatively very similar.

• Again key property is that real wages are inertial.

• Marginal likelihood of this sticky wage model is about 3 log points higher than AOB model.

• Conclude that performance of sticky wage model depends very much on troubling wage indexation assumption.
Unemployment Benefits

- Price setting frictions, monetary policy play a key role in determining the response of the economy to a change in $D$.

- In normal times, rise in $D$ increases the value of being unemployed.
  - Real wage rises, aggregate economic activity falls, unemployment rate rises.

- When ZLB is binding rise in $D$ gives rise to countervailing expansionary forces.
  - If those forces are sufficiently strong, could get an economic expansion.

- Given estimates about how long ZLB expected to last, AOB model implies that effects of increase in $D$ probably quite small.
Intuition, normal times

• In normal times, a rise in D increases the value of unemployment so that real wages rise.

• Reduces incentive of firms to post vacancies (standard).

• Rise in real wage leads to rise in inflation.

• Estimated monetary policy rule implies nominal interest rate rises by more than inflation.

• Rise in real interest rate drives spending on goods and services down.

  – Magnifies decline in aggregate economic activity.
• Standard contractionary effect - which raises the real wage and reduces firms’ incentive to post vacancies - is still present.
• Also increase in real wages leads continues to generate rise in inflation.

• With fixed nominal interest rate rise in inflation leads to fall in real interest rate.

• Drives spending on goods and services up.

• These expansionary forces are stronger the longer the ZLB is expected to bind relative to the duration of the increase in unemployment benefits.
AOB and Unemployment Benefits

Figure 8: Dynamic Effects of a Rise in Unemployment Benefits

Normal Times

1 Year ZLB

2 Years ZLB

Notes: 1 pp rise in unemployment benefits relative to steady state wage. Normal Times: Taylor rule. 1 or 2 Years ZLB: 1 or 2 years constant nominal interest rate.
We constructed a model that accounts for economy’s response to various business cycle shocks.

Our model captures key features of real wages: inertia.

- Allows us to account for weak response of inflation and strong responses of quantity variables to business cycle shocks.

We derive inertial wages from our specification of how firms and workers interact when negotiating wages.

This allows us to address policy questions that NK sticky wage models and reduced form real wage models can’t address.